COMPOSITE CONSTRUCTION

EXAMPLE 14.1 A precast pre-tensioned beam of rectangular section has a breadth of 100 mm and a depth of 200 mm. The beam, with an effective span of 5 m, is prestressed by tendons with their centroids coinciding with the bottom kern. The initial force in the tendons is 150 kN. The loss of prestress may be assumed to be 15 per cent. The beam is incorporated in a composite T-beam by casting a top flange of breadth 400 mm and thickness 40 mm. If the composite beam supports a live load of 8 kN/m^2, calculate the resultant stresses developed in the precast and *in situ* cast concrete assuming the pre-tensioned beam as: (a) unpropped, and (b) propped during the casting of the slab. Assume the same modulus of elasticity for concrete in precast beam and *in situ* cast slab.

Section properties of the pre-tensioned beam

$$A = (100 \times 200) = 20000 \text{ mm}^2$$

$$Z = \frac{(100 \times 200^2)}{6} = 667 \times 10^3 \text{ mm}^3$$

Initial prestressing force, $P = 150$ kN

Stresses due to prestressing force $= \left(\frac{2P}{A}\right)$

$= \left[\frac{(2 \times 150 \times 10^3)}{(20000)}\right] = 15$ N/mm^2 at the bottom and zero at the top fibre respectively.

Effective prestress after losses = $(0.85 \times 15) = 12.8\ N/mm^2$
Self-weight of the precast beam = $(0.1 \times 0.2 \times 24 \times 10^3) = 480\ N/m$
Self-weight moment = $(0.125 \times 480 \times 5^2) = 1500\ Nm$

Stresses at top and bottom fibre = $\dfrac{(1500000)}{(667 \times 10^3)} = \pm 2.25\ N/mm^2$

Self-weight of *in situ* cast slab = $(0.04 \times 0.4 \times 24 \times 10^3) = 384\ Nm$
Moment due to slab-weight = $(0.125 \times 384 \times 5^2) = 1200\ Nm$

Stresses due to slab-weight in the precast section = $\dfrac{(1200000)}{(667 \times 10^3)} = \pm 1.8\ N/mm^2$

Section properties of the composite section
Distance of the centroid from the top fibre = 87 mm

Second moment of area, $I = (1948 \times 10^5)\ mm^4$
Second moduli, $Z_t = (225 \times 10^4)\ mm^3$
$Z_b = (128 \times 10^4)\ mm^3$

Live load on the composite section = $(0.4 \times 1 \times 8000) = 3200$ N/m
Maximum live-load moment = $(0.125 \times 3200 \times 5^2) = 10000$ Nm
Live load stresses in the composite section

$$\text{At top} \quad = \left(\frac{10^7}{225} \times 10^4\right) = 4.45 \text{ N/mm}^2 \text{ (compression)}$$

$$\text{At bottom} \quad = \left(\frac{10^7}{128} \times 10^4\right) = 7.85 \text{ N/mm}^2 \text{ (tension)}$$

If the pre-tensioned beam is propped, the self-weight of the slab acts on the composite section.
Moment due to slab-weight = 1200 Nm
Stresses due to this moment in the composite section

$$\text{At top} \quad = \left(\frac{1200000}{225 \times 10^4}\right) = 0.53 \text{ N/mm}^2 \text{ (compression)}$$

$$\text{At bottom} \quad = \left(\frac{1200000}{128 \times 10^4}\right) = 0.94 \text{ N/mm}^2 \text{ (tension)}$$

The distribution of stresses for the various stages of loading for the propped and unpropped construction is shown in Fig. 14.3.

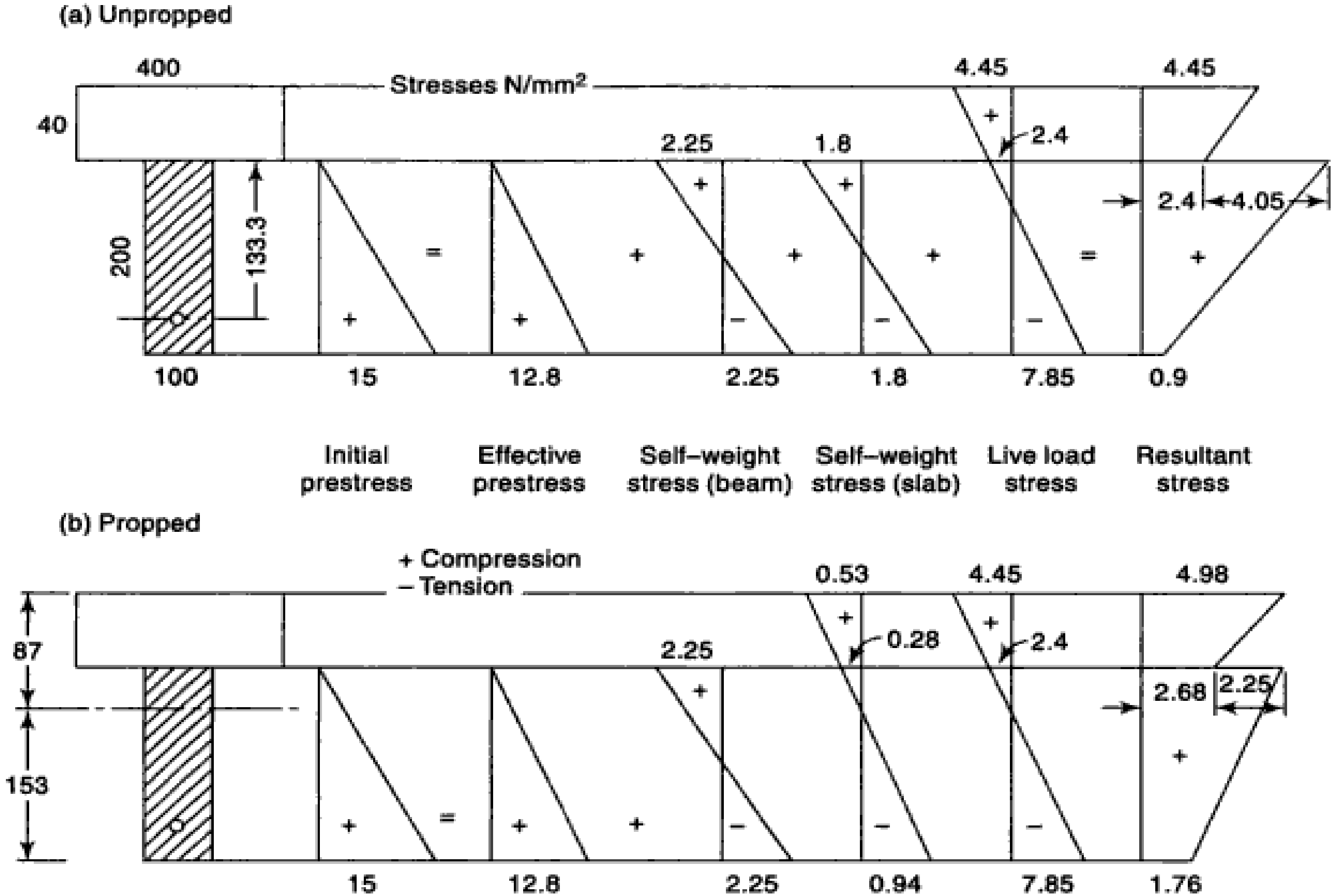

Fig. 14.3 *Stresses in Precast Pre-tensioned Beam and Cast in situ Slab*

EXAMPLE 14.2 Compute the resultant stresses developed in the precast pre-tensioned beam and cast *in situ* slab for the unpropped case if the modulus of elasticity of concrete in slab and beam are different. Assume E_c (prestressed beam) = 35 kN/mm^2.

$$\text{Ratio of modulus of elasticity} = \left(\frac{35}{28}\right) = 1.25$$

Properties of equivalent composite section

Area of *in situ* slab = $(400 \times 40) = 16000$ mm^2

Area of prestressed beam = $(200 \times 100) = 20000$ mm^2

The centroid of the equivalent composite section is determined by taking moments about an axis passing through the soffit of the beam.

If y = distance of the centroid from the soffit,

$$(16 + 1.25 \times 20)\, 10^3 \times y = (16 \times 10^3 \times 220) + (1.25 \times 20 \times 10^3 \times 100)$$

$$\therefore \quad y = 146 \text{ mm}$$

Second moment of area of the equivalent composite section is given by,

$$I_e = \left(\frac{400 \times 40^3}{12} + 16 \times 10^3 \times 74^2\right) + 1.25\left(\frac{100 \times 200^3}{12} + 20 \times 10^3 \times 46^2\right)$$

$$= 226 \times 10^6 \text{ mm}^4.$$

Live Load Moment = 10^7 N mm

Stresses developed in cast in situ slab

At the top of slab $= \dfrac{(10^7 \times 94)}{(226 \times 10^6)} = +4.15 \text{ N/mm}^2$

At the bottom of slab $= \dfrac{(10^7 \times 54)}{(226 \times 10^6)} = +2.2 \text{ N/mm}^2$

Stresses developed in the pre-tensioned beam

$$\text{At top} = \left[\frac{(10^7 \times 54)}{(226 \times 10^6)}\right] \times 1.25 = +2.75 \text{ N/mm}^2$$

$$\text{At bottom} = \left[\frac{(10^7 \times 146)}{(226 \times 10^6)}\right] \times 1.25 = -8.1 \text{ N/mm}^2$$

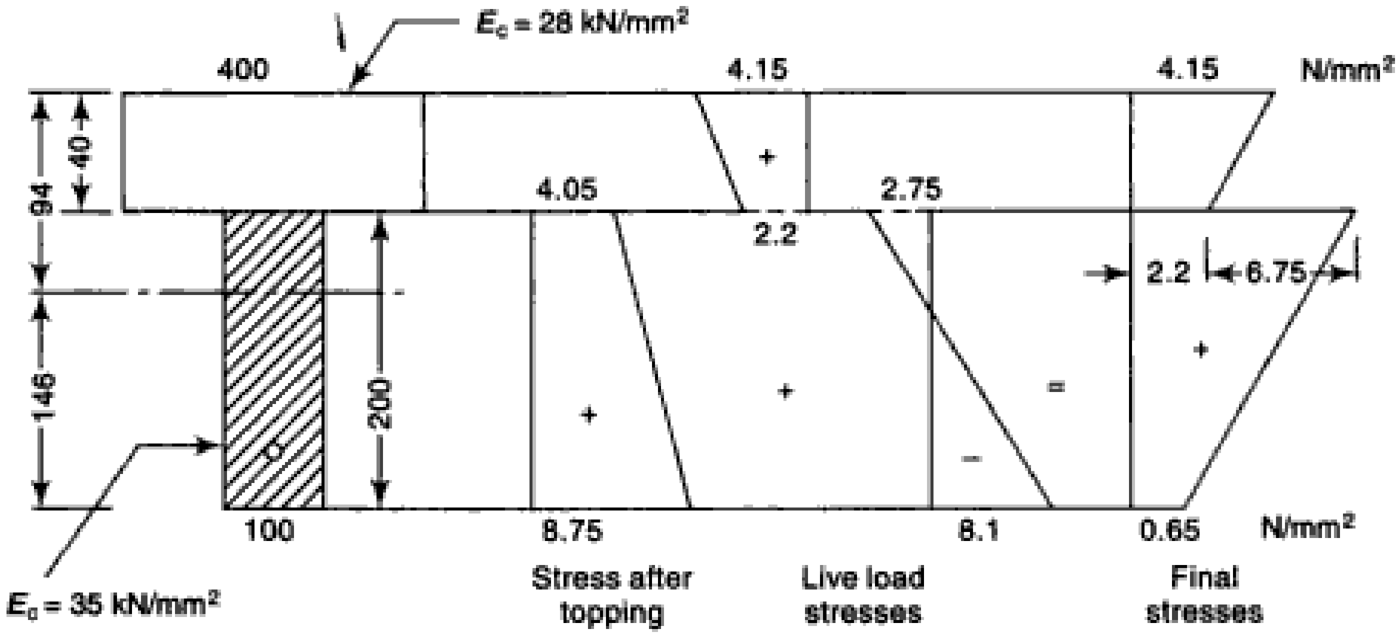

Fig. 14.4 *Stress Distribution*

EXAMPLE 14.3 A composite T-beam is made up of a pre-tensioned rib 100 mm wide and 200 mm deep, and a cast *in situ* slab 400 mm wide and 40 mm thick having a modulus of elasticity of 28 kN/mm². If the differential shrinkage is 100×10^{-6} units, determine the shrinkage stresses developed in the precast and cast *in situ* units.

Differential shrinkage, $\varepsilon_{cs} = 100 \times 10^{-6}$

Area of *in situ* concrete, $A_i = (400 \times 40) = 16000\ \text{mm}^2$

Uniform tensile stress induced in the cast *in situ* slab $= \varepsilon_{cs} E_c$

$= (100 \times 10^{-6})(28 \times 10^3) = 2.8\ \text{N/mm}^2$

Force, $N_{sh} = \varepsilon_{cs} E_c A_i = (100 \times 10^{-6})(28 \times 10^3)(16 \times 10^3) = (44.8 \times 10^3)\text{N}$

The centroid of the composite section is located 87 mm from the top fibre. Eccentricity of the compressive force, N_{sh}, from the centroid of the composite section = (87 – 20) = 67 mm

$\therefore$ Moment $= (44.8 \times 10^3) \times 67 = 3 \times 10^6$ N mm

Second moment of area of the composite section = (1948×10^5) mm^4

Section moduli for the various fibres:

Top fibre, Z_t $= (225 \times 10^4)$ mm^3

Bottom fibre, Z_b $= (128 \times 10^4)$ mm^3

Junction, Z_j $= (414 \times 10^4)$ mm^3

$$\text{Direct compressive stress} = \left[\frac{(44.8 \times 10^3)}{(36 \times 10^3)}\right] = 1.24 \text{ N/mm}^2$$

Bending stress:

$$\text{Top fibre} = \frac{(3 \times 10^6)}{(225 \times 10^4)} = 1.33 \text{ N/mm}^2$$

$$\text{Bottom fibre} = \frac{(3 \times 10^6)}{(128 \times 10^4)} = 2.34 \text{ N/mm}^2$$

$$\text{Junction} = \frac{(3 \times 10^6)}{(414 \times 10^4)} = 0.72 \text{ N/mm}^2$$

Differential shrinkage stresses:

(a) In precast pre-tensioned beam (+ compression – tension)

At the top of beam = (1.24 + 0.72) = 1.96 N/mm^2

At the bottom of beam = (1.24 – 2.34) = –1.10 N/mm^2

(b) In *in situ* cast slab,

At the top of slab = (1.24 + 1.33 – 2.8) = –0.23 N/mm^2

At the bottom of slab (junction) = (1.24 + 0.72 – 2.8) = –0.84 N/mm^2

The resultant shrinkage stress distribution is shown in Fig. 14.6.

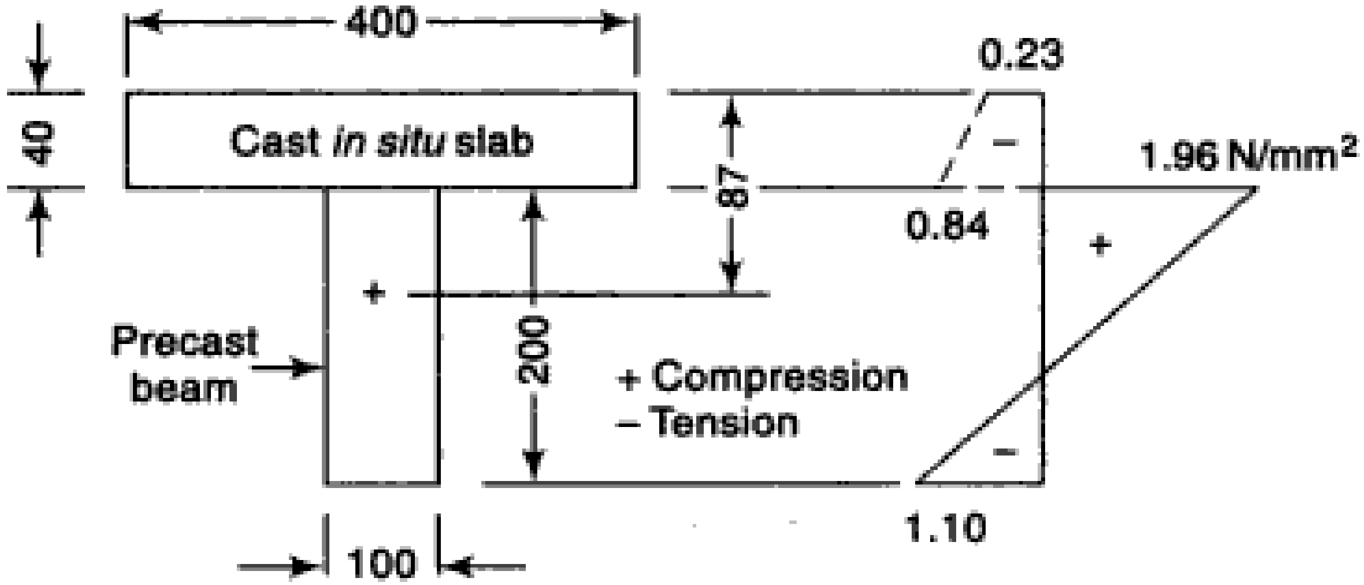

Fig. 14.6 *Stresses due to Differential Shrinkage*

Estimate for deflections

Example 14.5 A composite T girder of span 5 m is made up of a pre-tensioned rib, 100 mm wide by 200 mm deep, with an, *in situ* cast slab, 400 mm wide and 40 mm thick. The rib is prestressed by a straight cable having an eccentricity of 33.33 mm and carrying aninitial force of 150 kN. The loss of prestress may be assumed to be 15 per cent. Check the composite T-beam for the limit state of deflection if it supports an imposed load of 3.2 kN/m for: (a) unpropped construction, and (b) propped construction. Assume a modulus of elasticity of 35 kN/mm^2 for both precast and *in situ* cast elements.

Self-weight of precast beam = 0.48 N/mm

Self-weight of *in situ* cast slab = 0.384 N/mm

Imposed load on composite section = 3.2 N/mm

I for precast section = 667×105 mm^4

I for composite section = 1948×10^5 mm^4

Modulus of elasticity, $E = 35 \times 10^3\ \text{N/mm}^2$

Deflection due to prestress $= \left(\dfrac{PeL^2}{8EI}\right) = \left[\dfrac{150 \times 10^3 \times 33.33 \times 5000^2}{8 \times 35 \times 10^3 \times 667 \times 10^5}\right]$

$= 6.7$ mm (upward)

Effective deflection after losses $= (0.85 \times 6.7) = 5.7$ mm

Deflection due to self-weight of precast beam

$$\left(\frac{5gL^4}{384EI}\right) = \left[\frac{5 \times 0.48 \times 5000^4}{384 \times 35 \times 10^3 \times 667 \times 10^5}\right] = 1.7\ \text{mm}$$

Deflection of precast beam due to self-weight of cast *in situ* slab

$$= \left[\frac{(1.7 \times 0.384)}{(0.48)}\right] = 1.34\ \text{mm}$$

Deflection of composite beam due to live load

$$= \left[\frac{5 \times 3.2 \times 5000^4}{384 \times 35 \times 10^3 \times 1948 \times 10^5}\right] = 3.83\ \text{mm}$$

Deflection of composite beam due to self-weight of cast *in situ* slab

$$= \left(\frac{5 \times 0.384 \times 5000^4}{384 \times 35 \times 10^3 \times 1948 \times 10^5} \right) = 0.47 \text{ mm}$$

(a) Unpropped construction:

Resultant deflection under service loads = (–5.7 + 1.7+ 1.34 + 3.83)
= 1.17 mm

(b) Propped construction:

Resultant deflection under service loads = (–5.7 + 1.7 + 0.47 + 3.83)
= 0.30 mm

According to IS: 1343, the maximum permissible deflection under service loads is limited to a value of (span/250) = (5000/250) = 20 mm.

Flexural strength of composite members

EXAMPLE 14.7 The cross-section of a composite beam is of T-section having a pretensioned rib, 80 mm wide and 240 mm deep, and an *in situ* cast slab, 350 mm wide and 80 mm thick. The pre-tensioned beam is reinforced with eight wires of 5 mm diameter with an ultimate tensile strength of 1600 N/mm^2, located 60 mm from the soffit of the beam. The compressive strength of concrete in the *in situ* cast and precast elements is 20 and 40 N/mm^2 respectively. If adequate reinforcements are provided to prevent shear failure at the interface, estimate the flexural strength of the composite section.

$A_p = (20 \times 8) = 160\ \text{mm}^2$ $\qquad f_{ck} = 20\ \text{N/mm}^2$

$f_p = 1600\ \text{N/mm}^2$ $\qquad b = 350\ \text{mm} \quad d = 240\ \text{mm}$

The effective reinforcement ratio is given by

$$\left(\frac{f_p A_p}{f_{ck} bd}\right) = \left(\frac{1600 \times 160}{20 \times 350 \times 240}\right) = 0.152$$

Referring to Table 7.1,

$$\left(\frac{f_{pu}}{0.87 f_p}\right) = 1.0$$

$$\therefore \quad f_{pu} = (0.87 \times 1600) = 1392 \text{ N/mm}^2$$

$$\left(\frac{x_u}{d}\right) = 0.326$$

$$\therefore \quad x_u = (0.326 \times 240) = 78 \text{ mm}$$

Since x_u = 78 mm is less than the thickness of the flange (80 mm), the stress block is entirely within the *in situ* concrete. Hence, the flexural strength of the composite section is obtained as

$$M_u = f_{pu} A_p (d - 0.42 x_u)$$

$$= 1392 \times 160 \frac{(240 - 0.42 \times 78)}{10^6} = 46.15 \text{ kN m}$$

Shear strength of composite sections

EXAMPLE 14.9 A precast pre-tensioned rib, 80 mm wide and 240 mm deep, is to be connected to a M-25 grade *in situ* cast slab, 350 mm wide and 30 mm thick. Estimate the ultimate shearing force which will cause a separation of the two elements for the following two cases conforming to BS: 8110–1985 code specifications.

(a) If the surface is rough tamped and without links to withstand a horizontal shear stress of 0.6 N/mm^2, and

(b) With nominal links and the contact surfaces are as cast.

Assuming the moduli of elasticity of precast and *in situ* cast concrete to be equal, the centroid of the composite section is located 110 mm from the top of the slab. Second moment area of composite section = $I = (2487 \times 10^5)$ mm^4

Case (a)

If V_u = ultimate shearing force

$\tau = (V_u S/Ib)$

where $S = (350 \times 30 \times 95)$ mm^3

$b = 80$ mm

$$\tau = 0.6 \text{ N/mm}^2$$

$$\therefore \quad V_u = \left(\frac{0.6 \times 2487 \times 10^5 \times 80}{350 \times 30 \times 95} \right) = 12000 \text{ N} = 12 \text{ kN}$$

Case (b) When nominal links are provided and the contact surfaces are as cast, the design ultimate horizontal shear stress from Table 14.2 is obtained as

$$\tau = 1.2 \text{ N/mm}^2$$

Ultimate shear resistance is expressed as

$$V_u = \left(\frac{\tau I b}{S} \right)$$

$$= \left(\frac{1.2 \times 2487 \times 10^5 \times 80}{350 \times 30 \times 95} \right)$$

$$= 24{,}000 \text{ N} = 24 \text{ kN}$$