INTRODUCTION – THEORY AND BEHAVIOUR

Definition of Prestress:

- Prestress is defined as a method of applying pre-compression to control the stresses resulting due to external loads below the neutral axis of the beam tension developed due to external load which is more than the permissible limits of the plain concrete.

- The pre-compression applied (may be axial or eccentric) will induce the compressive stress below the neutral axis or as a whole of the beam c/s. Resulting either no tension or compression.

Basic Concept

- Prestressed concrete is basically concrete in which internal stresses of a suitable magnitude and distribution are introduced so that the stresses resulting from the external loads are counteracted to a desired degree.

Terminology

Tendon: A stretched element used in a concrete member of structure to impart prestress to the concrete. Generally high tensile steel wires, bars cables or strands used as tendons

Anchorage: A device generally used to enable the tendon to impart and maintain prestress in concrete.

MATERIALS FOR PRESTRESS CONCRETE MEMBERS

Concrete:

High grade of concrete. Prestress concrete requires concrete, which has a high compressive strength reasonably early age with comparatively higher tensile strength than ordinary concrete.

Steel:

High tensile steel , tendons , strands or cables

Necessity of high grade of concrete & steel:

- Higher the grade of concrete higher the bond strength which is vital in pretensioned concrete, Also higher bearing strength which is vital in post-tensioned concrete. Further creep & shrinkage losses are minimum with high-grade concrete.

- Generally minimum M30 grade concrete is used for post-tensioned & M40 grade concrete is used for pretensioned members.

- The losses in the prestress members due to various reasons generally in the range of 250 N/mm^2 to 400 N/mm^2. If mild steel or deformed steel are used the residual stresses may be zero or negligible. Hence high tensile steel wires are used which varies from 1600 to 2000 N/mm^2

SYSTEMS AND METHODS OF PRESTRESSING

We have a two systems of pre-stressing

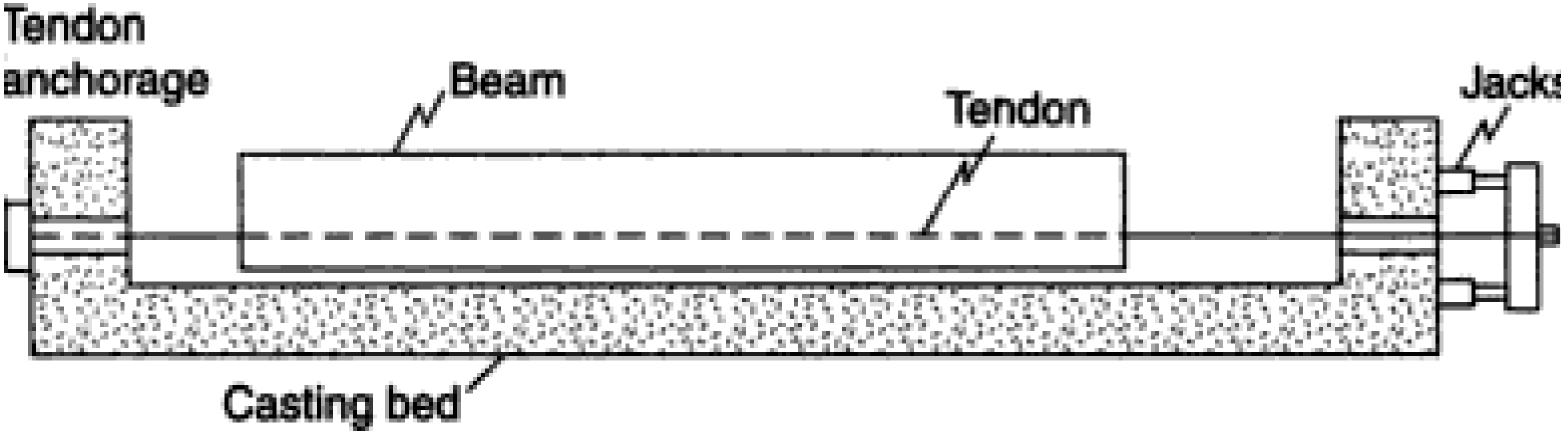
- Pre-tensioning
- Post-tensioning

Pre-tensioning: In which the tendons are tensioned before the concrete is placed, tendons are temporarily anchored and tensioned and the prestress is transferred to the concrete after it is hardened.

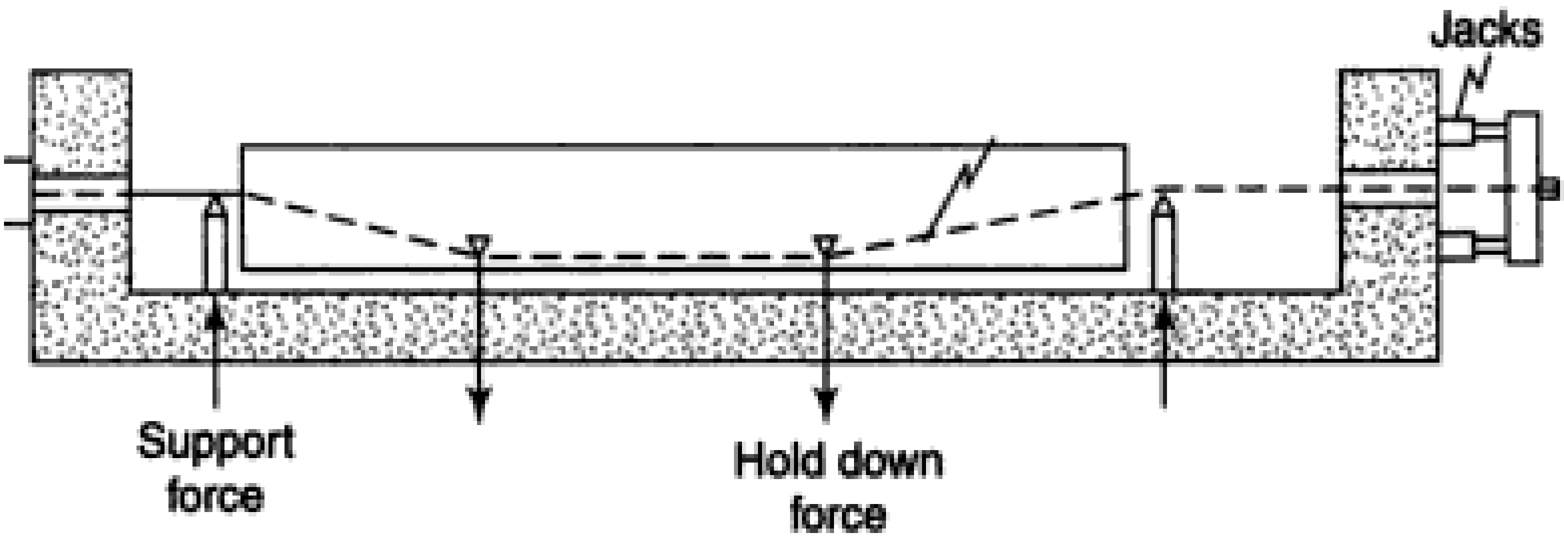
Post-tensioning: In which the tendon is tensioned after concrete has hardened. Tendons are placed in sheathing at suitable places in the member before casting and later after hardening of concrete.

Pretensioning system:

In the pre-tensioning systems, the tendons are first tensioned between rigid anchor-blocks cast on the ground or in a column or unit –mould types pretensioning bed, prior to the casting of concrete in the mould. The tendons comprising individual wires or strands are stretched with constant eccentricity or a variable eccentricity with tendon anchorage at one end and jacks at the other. With the forms in place, the concrete is cast around the stressed tendon.



(a) Beam with straight tendon



(b) Beam with variable tendon eccentricity

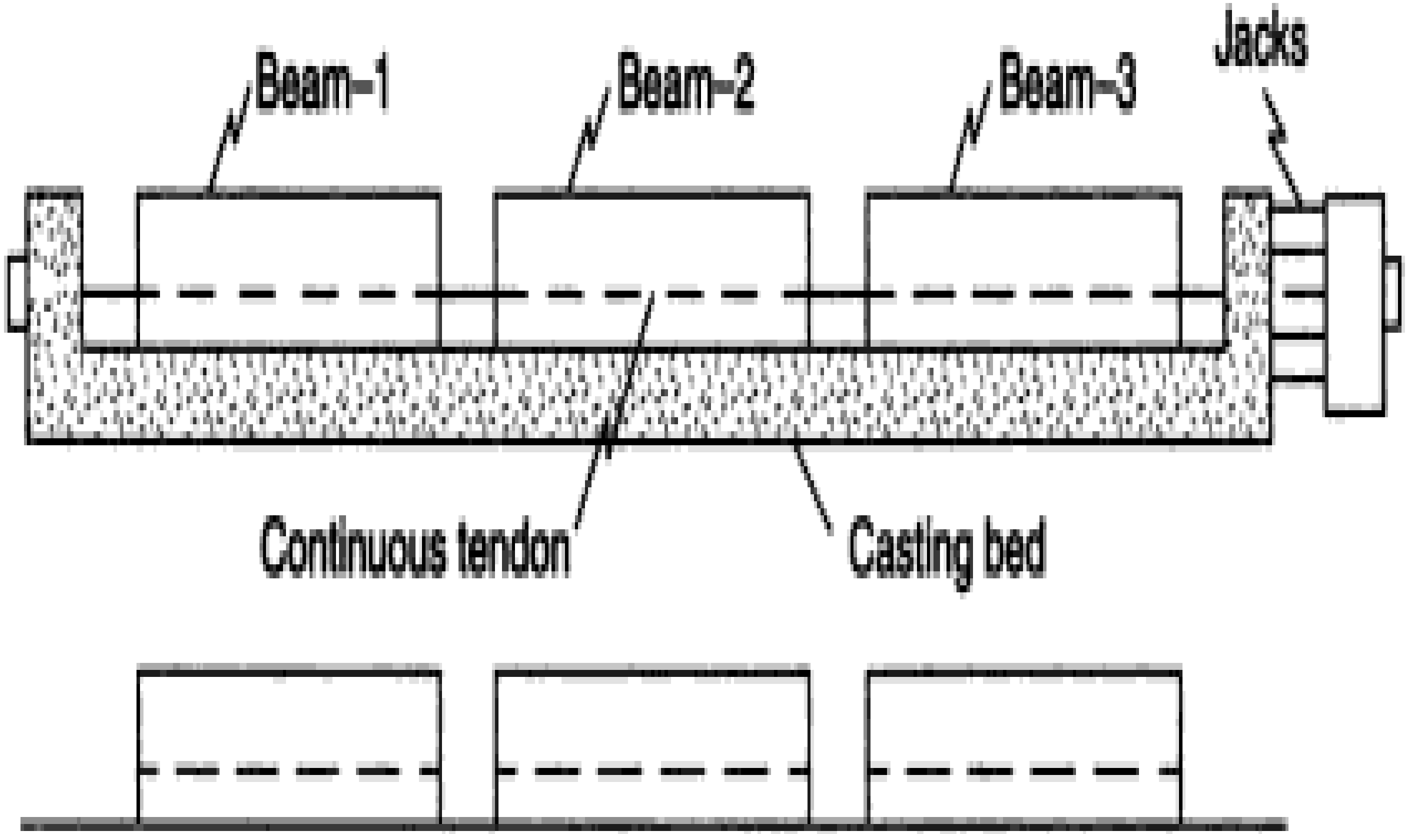


Fig. 3.2 *Hoyer's Long Line System of Pretensioning*

Pre-tensioned electric poles

Post-tensioned system:

- In post-tensioning the concrete unit are first cast by incorporating ducts or grooves to house the tendons. When the concrete attains sufficient strength, the high-tensile wires are tensioned by means of jack bearing on the end of the face of the member and anchored by wedge or nuts.

- The forces are transmitted to the concrete by means of end anchorage and, when the cable is curved, through the radial pressure between the cable and the duct. The space between the tendons and the duct is generally grouted after the tensioning operation.

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Most of the commercially patented prestressing systems are based on the following principle of anchoring the tendons:

1. Wedge action producing a frictional grip on the wire.
2. Direct bearing from the rivet or bolt heads formed at the end of the wire.
3. Looping the wire around the concrete.

Methos

1. Freyssinet system
2. Gifford-Udall system
3. Magnel blaton system
4. Lee-McCall system

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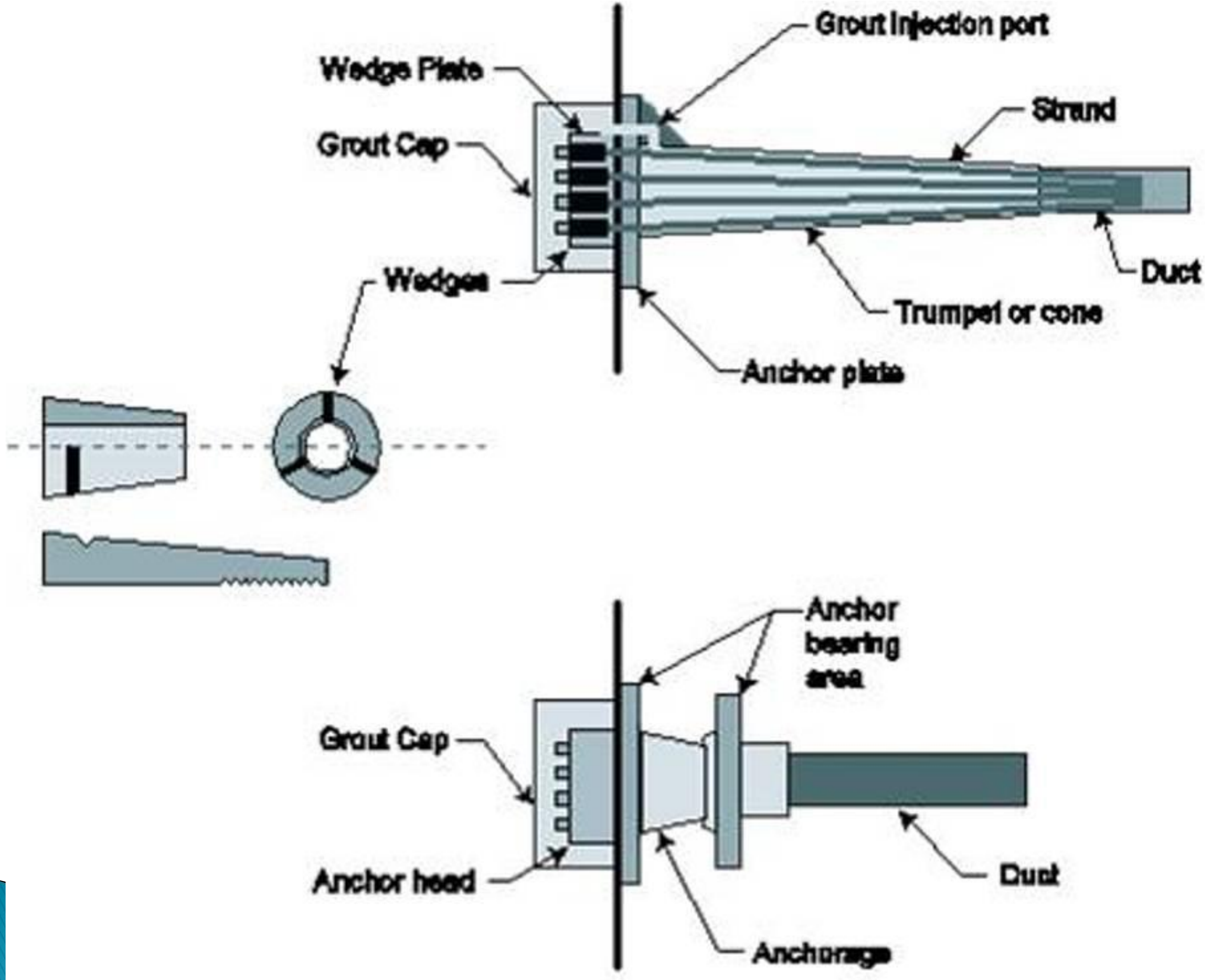


Grout injection port
Wedge Plate
Strand
Grout Cap
Duct
Wedges
Trumpet or cone
Anchor plate
Anchor bearing area
Grout Cap
Anchor head
Duct
Anchorage

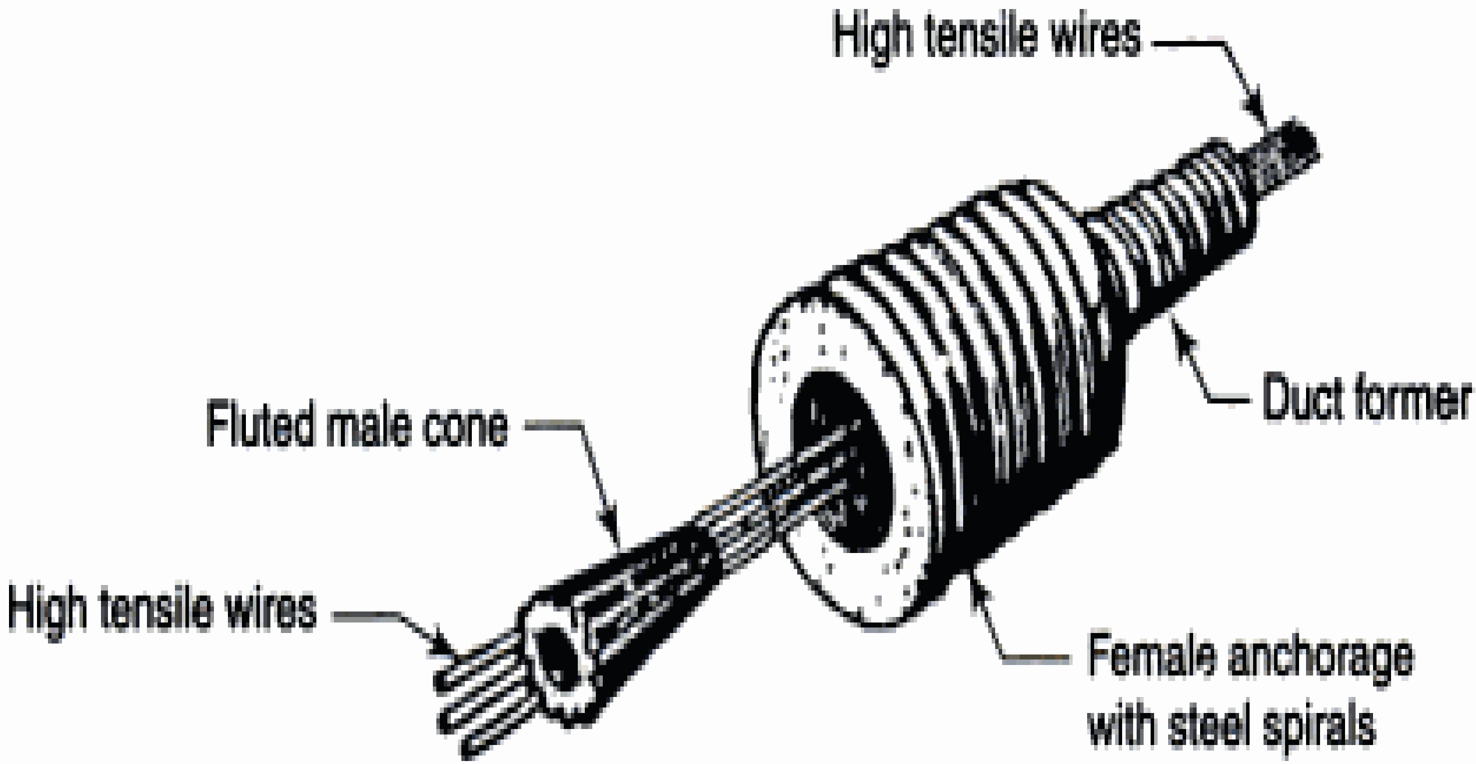


Fig. 3.4(a) *Freyssinet Anchorage*

Freyssinet system

Advantage of Prestressed Concrete

1) Section remains uncracked under service loads

Reduction of steel corrosion

- Increase in durability.

Full section is utilised

- Higher moment of inertia (higher stiffness)
- Less deformations (improved serviceability).
- Increase in shear capacity.
- Suitable for use in pressure vessels, liquid retaining structures.
- Improved performance (resilience) under dynamic and fatigue loading.

2) Suitable for precast construction The advantages of precast construction are as follows.

- Rapid construction
- Better quality control
- Reduced maintenance
- Suitable for repetitive construction
- Multiple use of formwork
- Reduction of formwork
- Availability of standard shapes.

ANALYSIS OF PRESTRESSED CONCRETE SECTION

Basic assumption

- Concrete is a homogenous material.

- Within the range of working stress, both concrete & steel behave elastically, notwithstanding the small amount of creep, which occurs in both the materials under the sustained loading.

- A plane section before bending is assumed to remain plane even after bending, which implies a linear strain distribution across the depth of the member.

Analysis of prestress member

The stress due to prestressing alone are generally combined stresses due to the action of direct load bending from an eccentrically applied load. The following notations and sign conventions are used for the analysis of prestress members.

P = Prestressing force (Positive when compressive)

e = Eccentricity of prestressing force

M = Pe= Moment

A = Cross-sectional area of the concrete member

I = Second moment of area of the section about its centroid

Zt & Zb = Section modulus of the top & bottom fibre respectively

Ftop & Fbot = Prestress in concrete developed at the top & bottom fibres

Yt & Yb = Distance of the top & bottom fibre from the centroid of the section

r= Radius of gyration

Concentric tendon

In this case, the load is applied concentrically and a compressive stress of magnitude (P/A) will act through out the section. Thus the stress will generate in the section as shown in the figure below.

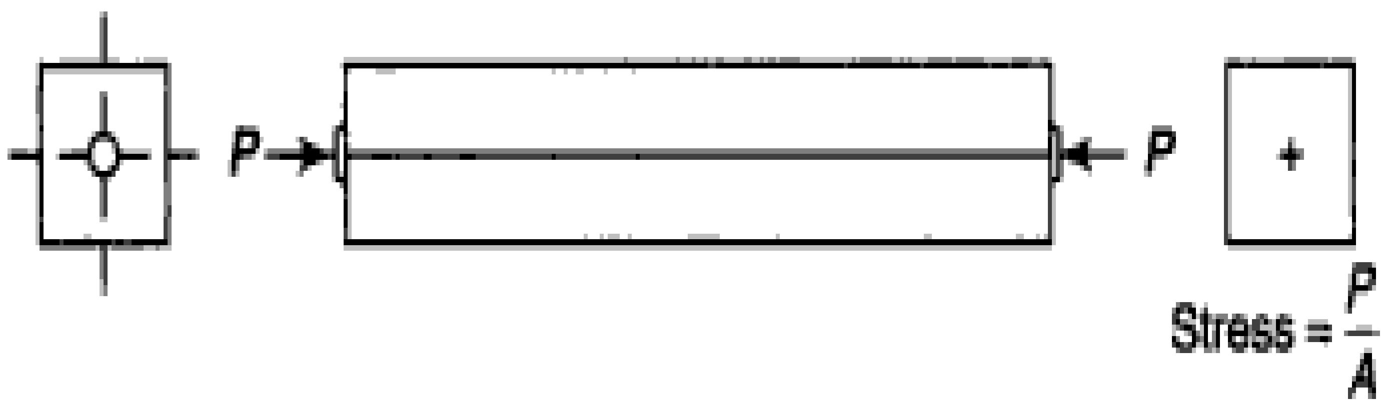


Fig. 4.1 *Concentric Prestressing*

Eccentric tendon

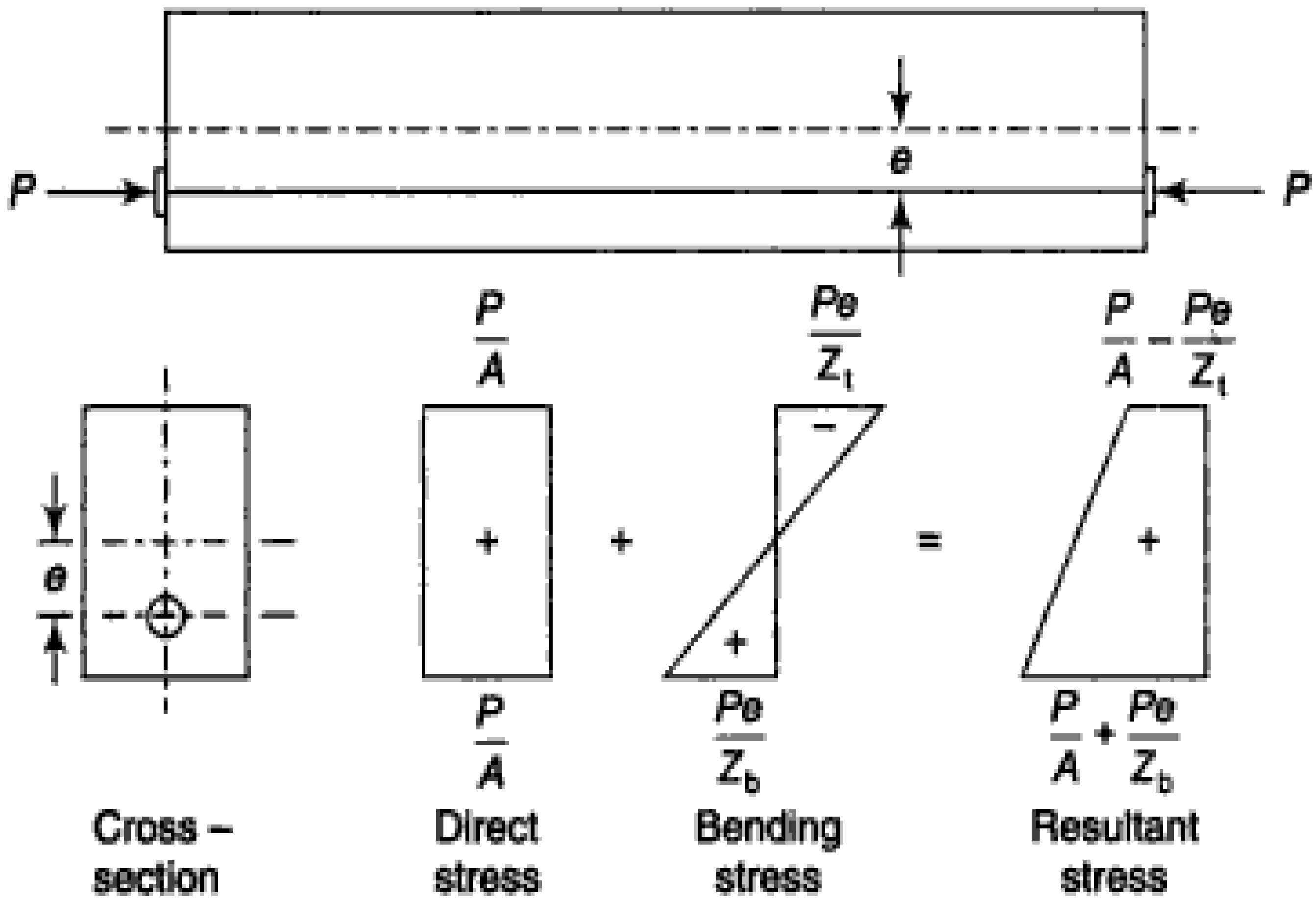
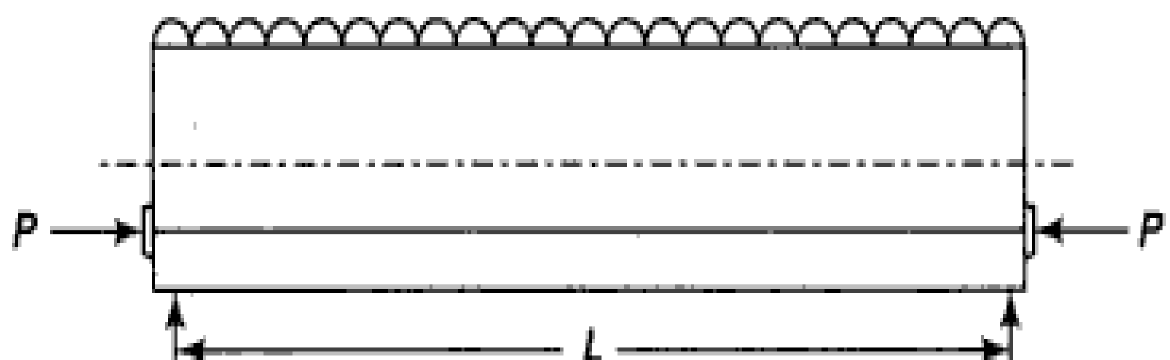


Fig. 4.2 *Eccentric Prestressing*

Resultant stress at a section

The concrete beam, shown in Fig. 4.3, supports uniformly distributed live and dead loads of intensity q and g. The beam is prestressed by a straight tendon carrying a prestressing force P at an eccentricity e. The resultant stresses in concrete at any section are obtained by superposing the effect of pretress and the flexural stresses developed due to the loads. If M_q and M_g are the live load and dead load moments at the central span section,

$$M_q = \left(\frac{qL^2}{8}\right) \quad M_g = \left(\frac{gL^2}{8}\right)$$



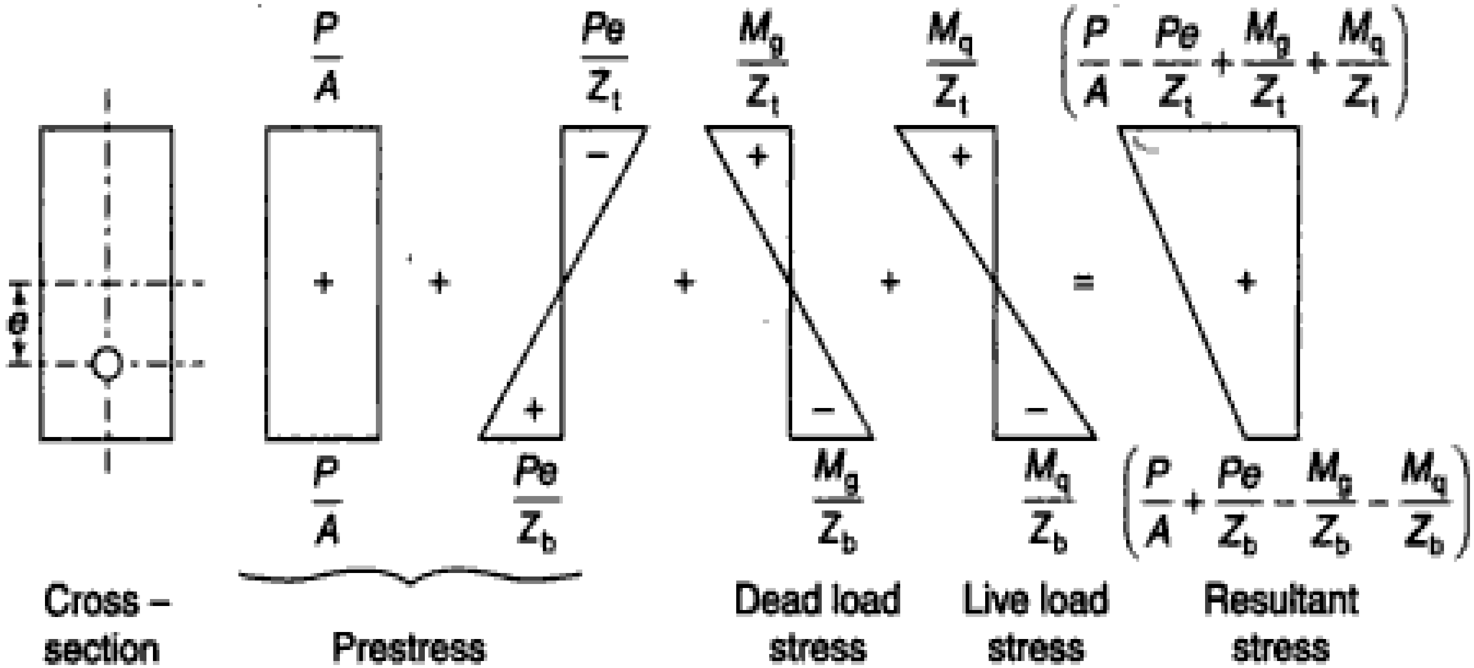


Fig. 4.3 *Stress Distribution due to Eccentric Prestressing, Dead and Live Loads*

The resultant stresses at the top and bottom fibres of concrete at any given section are obtained as:

$$f_{sup} = \left(\frac{P}{A} - \frac{Pe}{Z_t}\right) + \left(\frac{M_g}{Z_t}\right) + \left(\frac{M_q}{Z_t}\right)$$

$$f_{inf} = \left(\frac{P}{A} + \frac{Pe}{Z_b}\right) - \left(\frac{M_g}{Z_b}\right) - \left(\frac{M_q}{Z_b}\right)$$

EXAMPLE 4.2 A rectangular concrete beam of cross-section 30 cm deep and 20 cm wide is prestressed by means of 15 wires of 5 mm diameter located 6.5 cm from the bottom of the beam and 3 wires of diameter of 5 mm, 2.5 cm from the top. Assuming the prestress in the steel as 840 N/mm^2, calculate the stresses at the extreme fibres of the mid-span section when the beam is supporting its own weight over a span of 6 m. If a uniformly distributed live load of 6 kN/m is imposed, evaluate the maximum working stress in concrete. The density of concrete is 24 kN/m^3.

From Fig. 4.4,

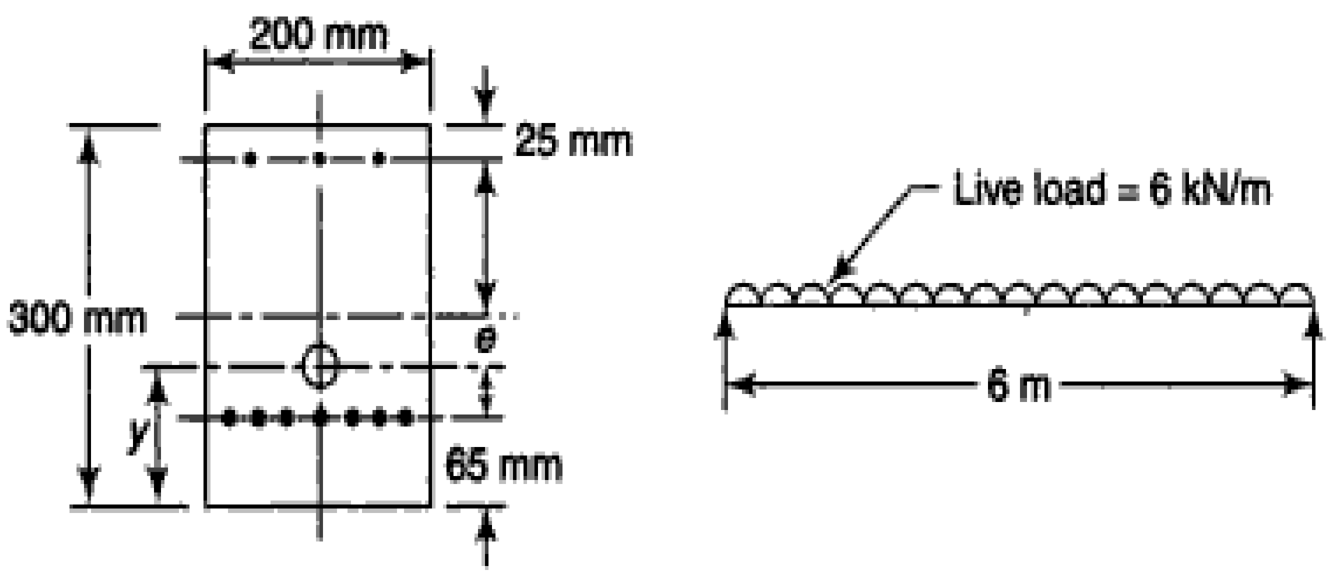


Fig. 4.4 *Prestressed Beam with Rectangular Section Supporting Live Loads*

Distance of the centroid of the prestressing force from the base,

$$y = \left[\frac{(15 \times 65) + (3 \times 275)}{18}\right] = 100 \text{ mm}$$

Eccentricity $e = (150 - 100) = 50$ mm

Prestressing force $P = (840 \times 18 \times 19.7) = 3 \times 10^5$ N

Area of cross-section $A = (300 \times 200) = 6 \times 10^4 \text{ mm}^2$

$$\text{Second moment of area } I = \left(\frac{200 \times 300^3}{12}\right) = 45 \times 10^7 \text{ mm}^4$$

$$\text{Section modulus } (Z_t \text{ and } Z_b) = \left(\frac{45 \times 10^7}{150}\right) = 3 \times 10^6 \text{ mm}^3$$

Self-weight of beam = $(0.3 \times 0.2 \times 24) = 1.44$ kN/m

Self-weight moment $M_g = \left(\frac{1.44 \times 6^2}{8}\right) = 6.48$ kNm

Live load moment $M_q = \left(\frac{6 \times 6^2}{8}\right) = 27$ kNm

Direct stress due to prestress $\left(\frac{P}{A}\right) = \left(\frac{3 \times 10^5}{6 \times 10^6}\right) = 5$ N/mm^2

Bending stress due to prestress $\left(\frac{Pe}{Z}\right) = \left(\frac{3 \times 10^5 \times 50}{3 \times 10^6}\right) = 5$ N/mm^2

Self-weight stress $M_g/Z = \left(\frac{6.48 \times 10^6}{3 \times 10^6}\right) = 2.16$ N/mm^2

Live load stress $M_q/Z = \left(\frac{27 \times 10^6}{3 \times 10^6}\right) = 9$ N/mm^2

The resultant stresses due to (self-weight + prestress + live load) are shown in Fig. 4.5.
Maximum working stress in concrete = 11.16 N/mm^2 (compression).

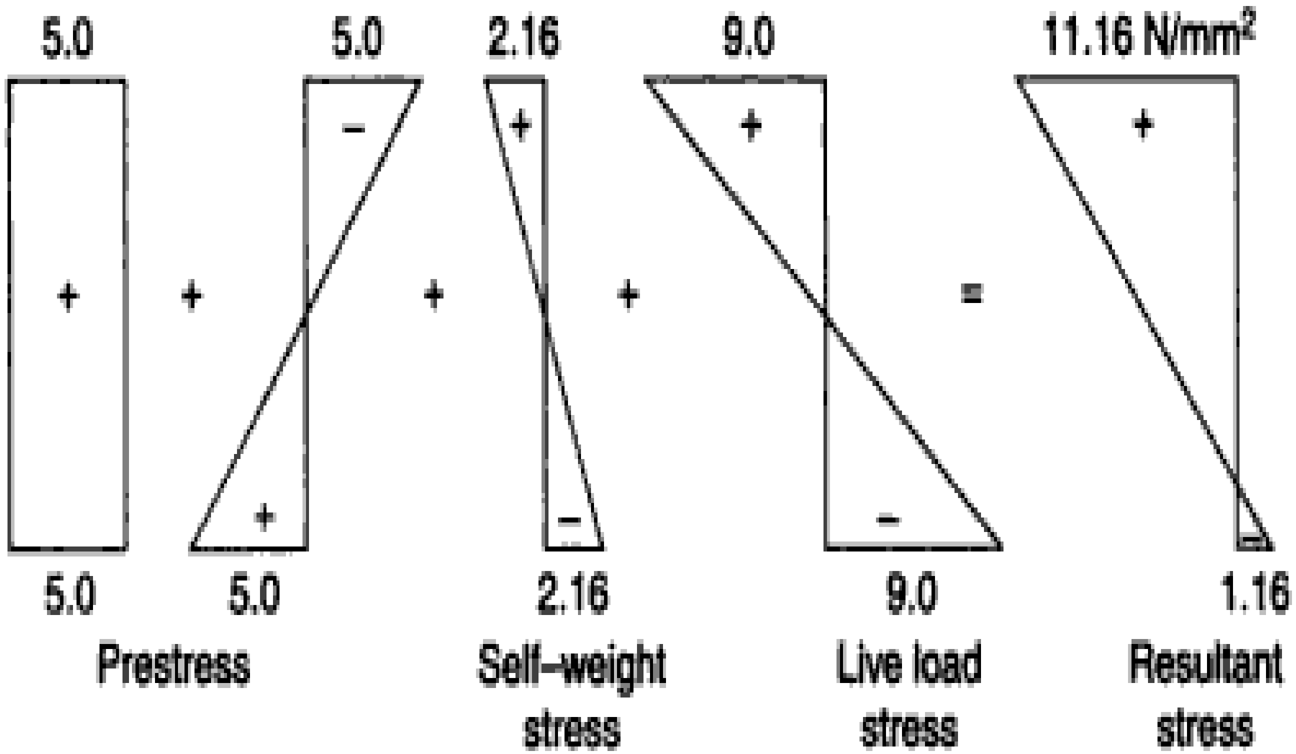


Fig. 4.5 *Analysis of Stresses at Mid-Span*

EXAMPLE 4.3 An unsymmetrical I-section beam is used to support an imposed load of 2 kN/m over a span of 8 m. The sectional details are top flange, 300 mm wide and 60 mm thick; bottom flang, 100 mm wide and 60 mm thick; thickness of the web = 80 mm; overall depth of the beam = 400 mm. At the centre of the span, the effective prestressing force of 100 kN is located at 50 mm from the soffit of the beam. Estimate the stresses at the centre of span section of the beam for the following load conditions:

(a) Prestress + self-weight
(b) Prestress + self-weight + live load.

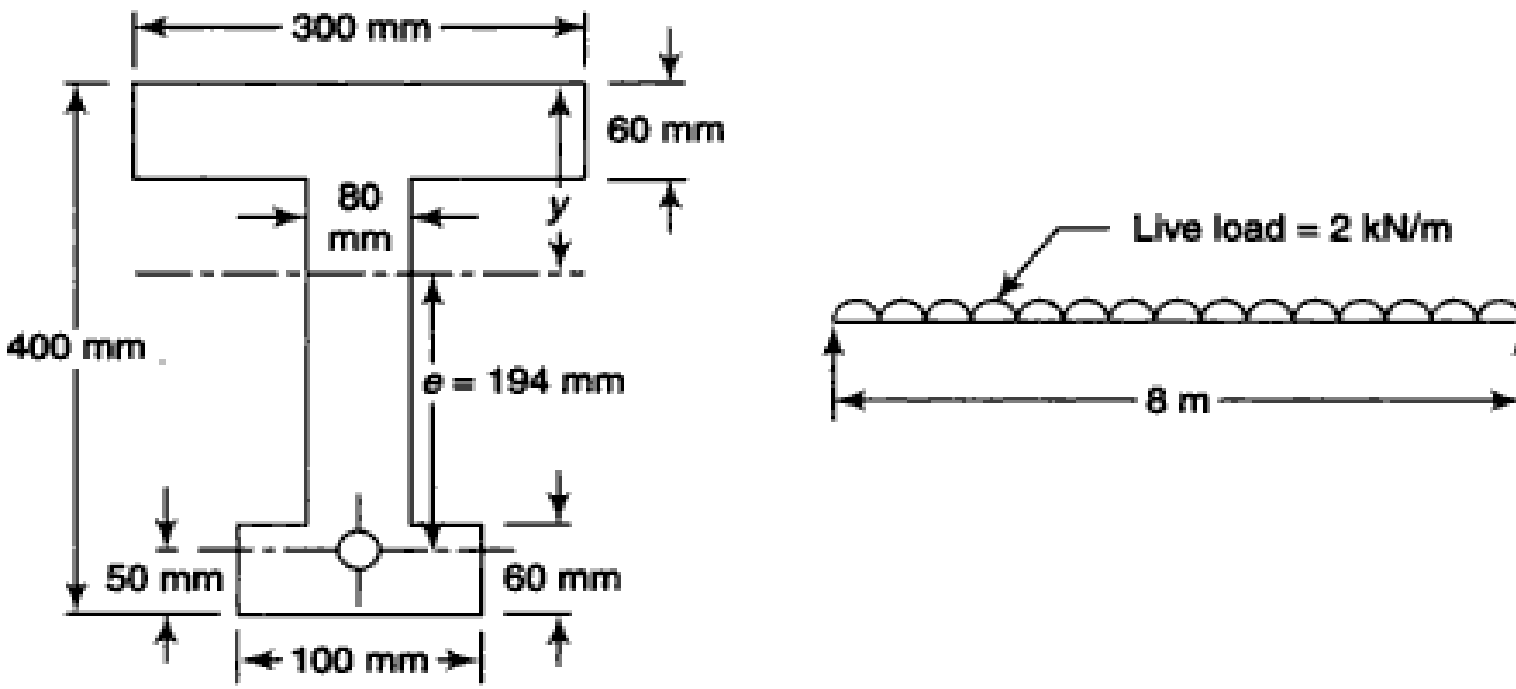


Fig. 4.6 *Prestressed Beam with Unsymmetrical I-Section Supporting Live Loads*

Stresses at the centre of span

Type of stress	*At top fibre* (N/mm^2)	*At bottom fibre* (N/mm^2)
Prestress	$P/A = +2.15$	$P/A = +2.15$
	$Pe/Z_t = -4.0$	$Pe/Z_b = +6.25$
Self-weight stress	$M_g/Z_t = +1.85$	$M_g/Z_b = -2.9$
Live load stress	$M_q/Z_t = +3.3$	$M_q/Z_b = -5.15$

+ Compression – Tension

Resultant stresses:

(a) (Prestress + self-weight stress) = 0, and + 5.5 N/mm^2

(b) Prestress + self-weight stress

+ live load stress) = + 3.3 N/mm^2, and + 0.35 N/mm^2.

EXAMPLE 4.5 A prestressed concrete beam of section 200 mm wide by 300 mm deep is used over an effective span of 6 m to support an imposed load of 4 kN/m. The density of concrete is 24 kN/m³.

At the centre of span section of the beam, fmd the magnitude of:

(a) the concentric prestressing force necessary for zero fibre-stress at the soffit when the beam is fully loaded; and

(b) the eccentric prestressing force located 100 mm from the bottom of the beam which would nullify the bottom fibre stresses due to loading.

$$A = (200 \times 300) = 6 \times 10^4 \text{ mm}^2$$

$$Z_b = Z_t = \left(\frac{200 \times 300^2}{6}\right) = 3 \times 10^6 \text{ mm}^3$$

$$g = (0.2 \times 0.3 \times 24) = 1.44 \text{ kN/m}$$

$$M_g = (0.125 \times 1.44 \times 6^2) = 6.48 \text{ kN m}$$
$$M_q = (0.125 \times 4 \times 6^2) = 18 \text{ kNm}$$

Tensile stress at the bottom fibre due to dead and live loads

$$= \left[\frac{(6.48 + 18)10^6}{3 \times 10^6}\right] = 8.16 \text{ N/mm}^2$$

(a) If P = concentric prestressing force, for zero stress at the soffit of the beam under loads

$$P/A = 8.16$$

$$\therefore \quad P = (8.16 \times 6 \times 10^4) = 489.6 \text{ kN}$$

(b) If P = eccentric prestressing force (e = 50 mm), for zero stress at the soffit of the beam under loads

$$(P/A) + (Pe/Z_b) = 8.16$$

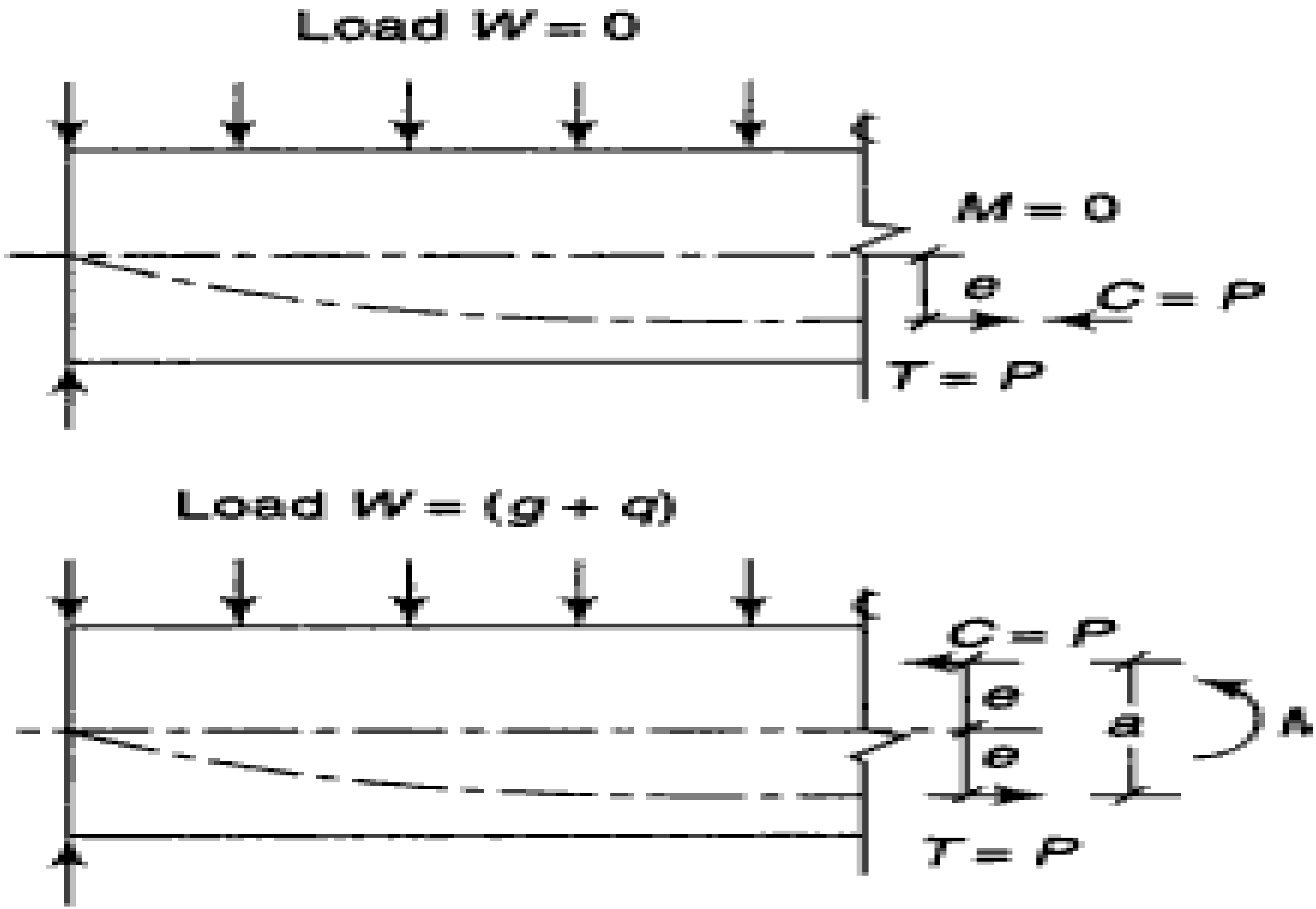
$$\therefore \quad P\left(\frac{1}{6 \times 10^4} + \frac{50}{3 \times 10^6}\right) = 8.16$$

$$\therefore \quad P = 244.8 \text{ kN}$$

The magnitudes of the computed prestressing forces clearly indicate the advantages of eccentric prestressing in flexural members subjected to transverse loads.

PRESSURE LINE OR THRUST LINE

- In prestress, the combined effect of prestressing force & external load can be resolved into a single force. The locus of the points of application of this force in any structure is termed as the pressure line or thrust line. The load here is such that stress at top fiber of support & bottom fiber of the central span is zero.



(b) Prestressed concrete beam

If M = bending moment at the section due to dead and live loads

e = eccentricity of the tendon

$T = P$ = prestressing force in the tendon

moment equilibrium yields the relation,

$$M = Ca = Ta = Pa \quad \text{and} \quad a = \left(\frac{M}{P}\right)$$

The shift of pressure line e measured from the centroidal axis is obtained as

$$e' = (a - e) = \left(\frac{M}{P}\right) - e$$

The resultant stresses at the top and bottom fibres of the section are expressed as,

$$f_{sup} = \left(\frac{P}{A}\right) + \left(\frac{Pe'}{Z_t}\right)$$

$$f_{inf} = \left(\frac{P}{A}\right) - \left(\frac{Pe'}{Z_b}\right)$$

EXAMPLE 4.7 A prestressed concrete beam of section 120 mm wide by 300 mm deep is used over an effective span of 6 m to support a uniformly distributed load of 4 kN/m, which includes the self-weight of the beam. The beam is prestressed by a straight cable carrying a force of 180 kN and located at an eccentricity of 50 mm. Determine the location of the thrust-line in the beam and plot its position at quarter and central span sections.

$$P = 180 \text{ kN}$$
$$E = 50 \text{ mm}$$
$$A = 36 \times 10^3 \text{ mm}^2$$
$$Z = 18 \times 10^5 \text{ mm}^3$$

Stresses due to prestressing force

$$P/A = \left(\frac{180 \times 10^3}{36 \times 10^3}\right) = +5 \text{ N/mm}^2$$

$$Pe/Z = \left(\frac{180 \times 10^3 \times 50}{18 \times 10^3}\right) = +5 \text{ N/mm}^2$$

Bending moment at the centre of the span = $(0.125 \times 4 \times 6^2) = 18$ kN m

Bendmg stresses at top and bottom = $\left(\dfrac{18 \times 10^6}{18 \times 10^5}\right) = \pm 10$ N/mm^2

Resultant stresses at the central section:

$$\text{At top} = (5 - 5 + 10) = 10 \text{ N/mm}^2$$
$$\text{At bottom} = (5 + 5 - 10) = 0 \text{ N/mm}^2$$

Shift of pressure-line from cable-line = $M/P = \left(\dfrac{18 \times 10^6}{18 \times 10^4}\right) = 100$ mm

Bending moment at quarter span section = $(3/32)\, qL^2 = (3/32) \times 4 \times 6^2$
$= 13.5$ kN m

Bendmg stress at top and bottom = $\left(\dfrac{13.5 \times 10^6}{18 \times 10^5}\right) = 7.5$ N/mm^2

Resultant stresses at the quarter span section:

$$\text{At top} = (5 - 5 + 7.5) = 7.5\ \text{N/mm}^2$$

$$\text{At bottom} = (5 + 5 - 7.5) = 2.5\ \text{N/mm}^2$$

Shift of pressure-line from cable-line $M/P = \left(\dfrac{13.5 \times 10^6}{18 \times 10^4}\right) = 75\ \text{mm}$

The location of pressure line is shown in Fig. 4.13.

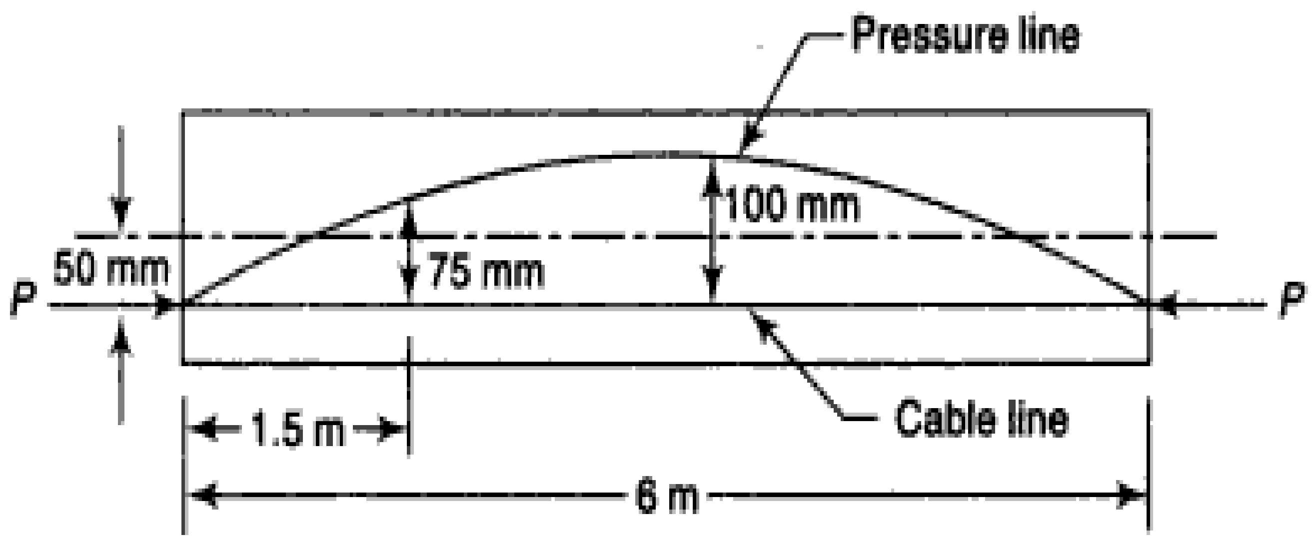


Fig. 4.13 *Location of Pressure Line in the Prestressed Beam*

Example 4.6 A prestressed concrete beam with a rectangular section 120 mm wide by 300 mm deep supports a uniformly distributed load of 4 kN/m, which includes the self-weight of the beam. The effective span of the beam is 6 m. The beam is concentrically prestressed by a cable carrying a force of 180 kN. Locate the position of the pressure line in the beam.

Prestressing force, $P = 180$ kN

Eccentricity, $e = 0$

$A = 36 \times 10^3 \text{ mm}^2,\ Z_t = Z_b = 18 \times 10^5 \text{ mm}^3$

Bending moment at the centre of the span $= (0.125 \times 4 \times 6^2) = 18$ kNm

$$\text{Direct stress } \frac{P}{A} = \left(\frac{180 \times 10^3}{36 \times 10^3}\right) = 5 \text{ N/mm}^2$$

$$\text{Bending stress } \frac{M}{Z} = \left(\frac{18 \times 10^6}{18 \times 10^5}\right) = 10 \text{ N/mm}^2$$

Resultant stresses at the centre of the span section:

At top = (5 + 10) = 15 N/mm² (Compression)

At bottom = (5 – 10) = –5 N/mm² (Tension)

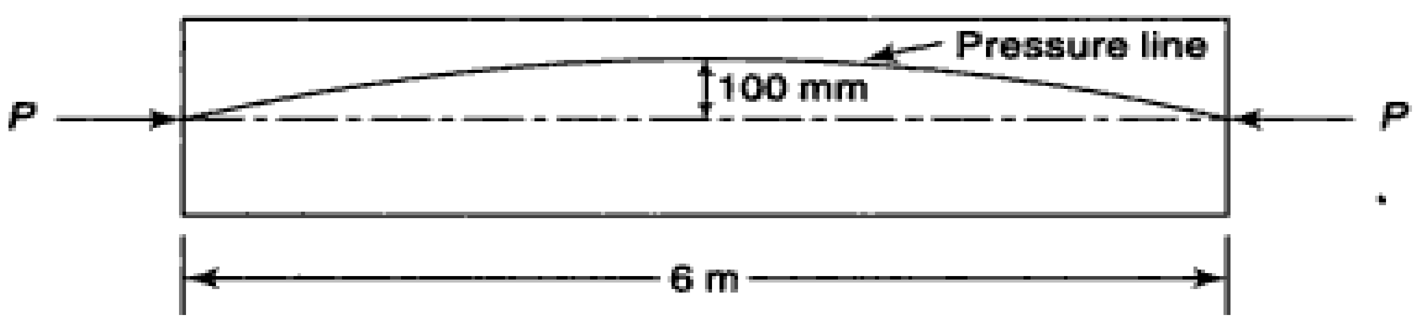
If N = resultant thrust in the section, and e = corresponding eccentricity (shift of pressure line), then,

$$N/A + Ne/Z = 15$$

$$\text{But } N = 180 \times 10^3 \text{ N}$$

$$A = 36 \times 10^3 \text{ mm}^2$$

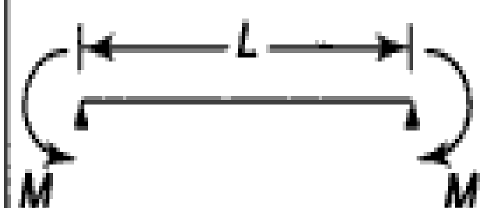
(solving, e = 100 mm)

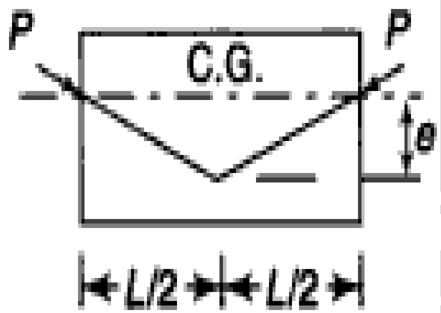
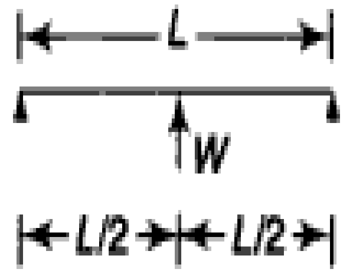
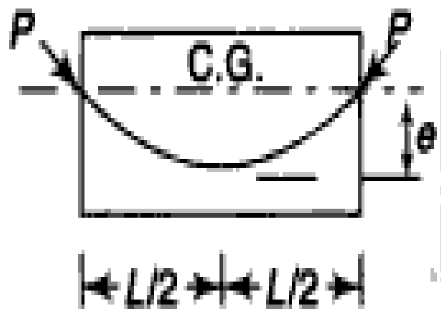
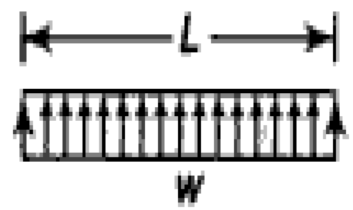
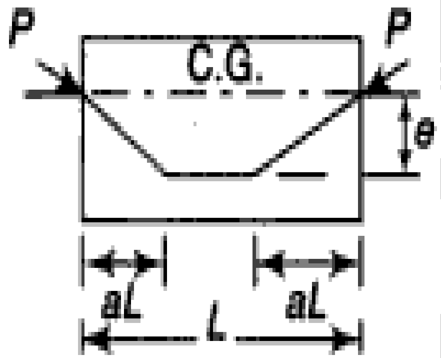
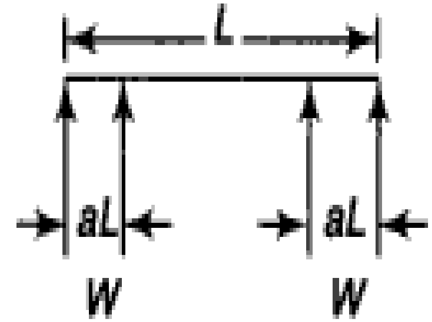


LOAD BALANCING CONCEPT

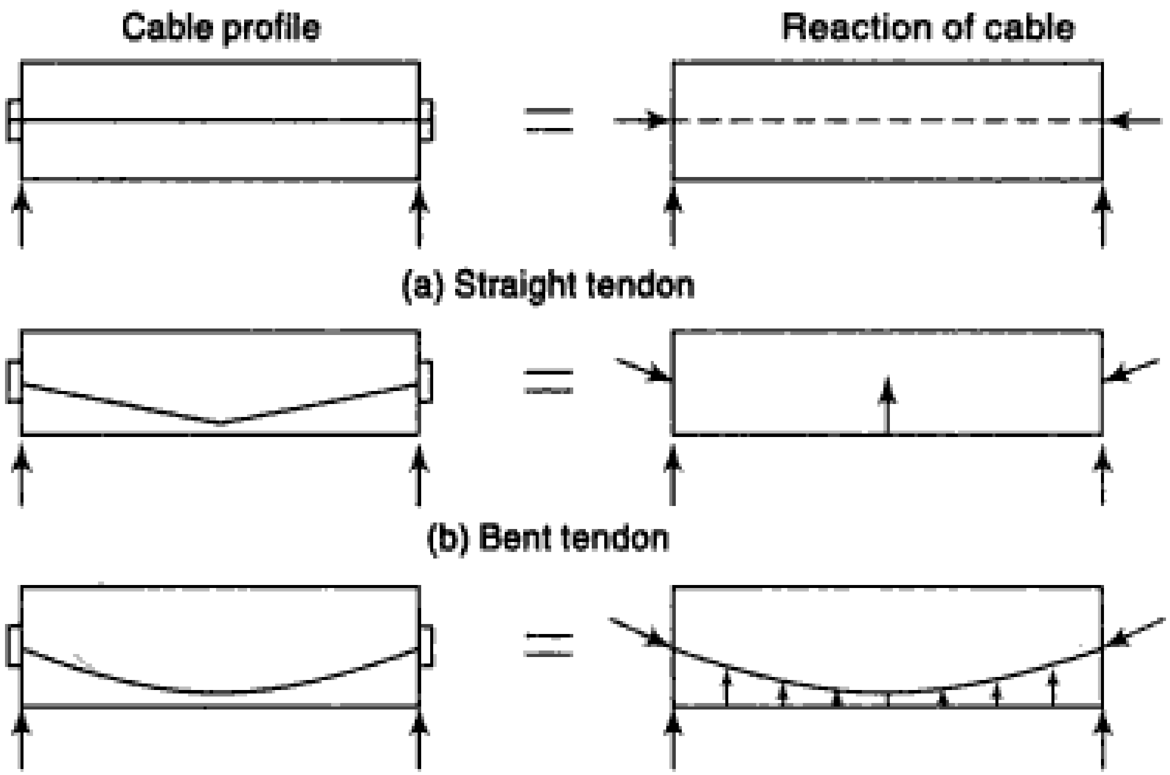
It is possible to select suitable cable profiles in a prestressed concrete member such that the transverse component of the cable force balances the given type of external loads. This can be readily illustrated by considering the free-body of concrete, with the tendon replaced by forces acting on the concrete beam as shown in Fig. 4.18 and Table 4.1.

Table 4.1 *Tendon Profiles and Equivalent Loads in Prestressed Concrete Beams*

Tendon profile	*Equivalent moment or Load*	*Equivalent loading*	*Camber*
	$M = Pe$		$\frac{ML^2}{8EI}$

	$W = \frac{4Pe}{L}$		$\frac{WL^3}{48EI}$
	$W = \frac{8Pe}{L^2}$		$\frac{5WL^4}{384EI}$
	$W = \frac{Pe}{aL}$		$\frac{a(3-4a^2)WL^3}{24EI}$

In general, this requirement will be satisfied if the cable profile in a prestressed member corresponds to the shape of the bending moment diagram resulting from the external loads. Thus, if the beam supports two concentrated loads, the cable should follow a trapezoidal profile. If the beam supports uniformly distributed loads, the corresponding tendon should follow a parabolic profile. The principle of load-balancing is further amplified with the following examples.



(c) Curved tendon

EXAMPLE 4.12 A rectangular concrete beam 300 mm wide and 800 mm deep supports two concentrated loads of 20 kN each at the third point of a span of 9 m.

(a) Suggest a suitable cable profile. If the eccentricity of the cable profile is 100 mm for the middle third portion of the beam, calculate the prestressing force required to balance the bending effect of the concentrated loads (neglect the self-weight of the beam).

(b) For the same cable profile, find the effective force in the cable if the resultant stress due to self-weight, imposed loads and prestressing force is zero at the bottom fibre of the mid-span section.

(Assume $D_c = 24\ kN/m^3$)

(a) A trapezoidal cable profile is selected since the bending moment diagram due to the two concentrated loads is trapezoidal in shape.

Given $Q = 20$ kN, $e = 100$ mm, $L = 9$ m, $Z = 32 \times 10^6\ mm^3$,

P = Prestressing force

$$Pe = \left(\frac{QL}{3}\right) \quad \therefore P = \left(\frac{QL}{3e}\right) = \left(\frac{20 \times 9000}{3 \times 100}\right) = 600 \text{ kN}$$

(b) Self-weight of the beam, $g = (0.3 \times 9.8 \times 24) = 5.76$ kN/m

Self-weight moment, $M_g = (0.125 \times 5.76 \times 9^2) = 58.32$ kN m

$$\text{Bending stress} = \left(\frac{58.32 \times 10^6}{32 \times 10^6}\right) = 1.82 \text{ N/mm}^2$$

$$\text{Moment at the centre due to loads} = \left(\frac{QL}{3}\right) = \frac{(20 \times 9)}{3} = 60 \text{ kN m}$$

$$\text{Stresses due to loads} = \left(\frac{60 \times 10^6}{32 \times 10^6}\right) = 1.875 \text{ N/mm}^2$$

Total tensile stress at the bottom fibre = (1.82 + 1.875) = 3.695 N/mm^2

If P = required prestressing force in the cable,

$$e = 100 \text{ mm}$$

$$A = (300 \times 800) = 24 \times 10. \text{ mm}^2.$$

$$\left(\frac{P}{A} + \frac{Pe}{Z}\right) = 3.695$$

$$P\left(\frac{1}{24} \times 10^4 + \frac{100}{32 \times 10^6}\right) = 3.695$$

$\therefore$ $P = 507 \times 10^3 \text{ N} = 507 \text{ kN}$

EXAMPLE 4.13 A prestressed concrete beam supports an imposed load of 4 kN/m over an effective span of 10 m. The beam has a rectangular section with a width of 200 mm and depth of 600 mm. Find the effective prestressing force in the cable if it is parabolic with an eccentricity of 100 mm at the centre and zero at the ends, for the following conditions:

(a) if the bending effect of the prestressing force is nullified by the imposed load for the mid-span section (neglecting self weight of beam).

(b) if the resultant stress due to self-weight, imposed load and prestressing force is zero at the soffit of the beam for the mid-span section (assume $D_c = 24$ kN/m³).

$$A = 12 \times 10^4 \text{ mm}^2, \qquad e = 100 \text{ mm},$$

$$Z = 12 \times 10^6 \text{ mm}^3, \qquad q = 4 \text{ kN/m}$$

Self-weight of the beam, $g = (0.2 \times 0.6 \times 24) = 2.88$ kN/m

(a) If P = prestressing force, $Pe = (qL^2/8)$

$$\therefore \qquad P = \left(\frac{qL^2}{8e}\right) = \left(\frac{4 \times 10^2}{8 \times 0.1}\right) = 500 \text{ kN}$$

(b) Total load on beam = (2.88 + 4.00) = 6.88 kN/m
Bending moment at the centre of the span,

$$M = (0.125 \times 6.88 \times 10^2) = 86 \text{ kNm}$$

For the bottom fibre stress to be zero,

$$\left(\frac{P}{A} + \frac{Pe}{Z}\right) = \frac{M}{Z}$$

$$\left[\left(\frac{P}{12 \times 10^4}\right) + \left(\frac{P \times 100}{12 \times 10^6}\right)\right] = \frac{(86 \times 10^6)}{(12 \times 10^6)}$$

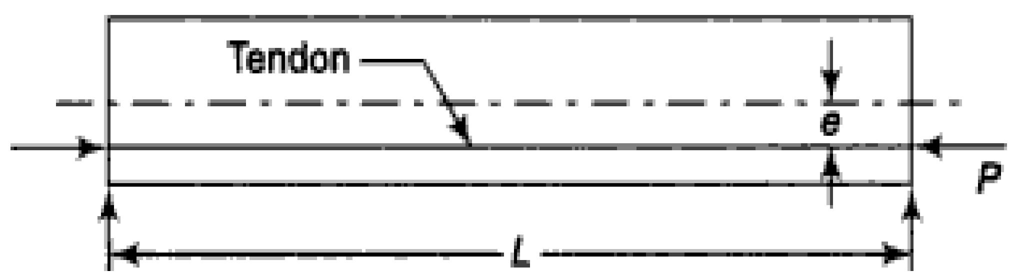
$$P = 430 \times 10^3 \text{ N} = 430 \text{ kN}$$

EFFECT OF LOADING ON TENSILE STRESSES IN TENDONS

A prestressed member undergoes deformation due to the action of the prestressing force and transverse loads acting on the member. Consequently, the curvature of the cable changes, which results in a slight variation of stresses in the tendons. Considering Fig. 4.21, in which a concrete beam of span L is prestressed by a cable carrying an effective force P at an eccentricity, e the rotation θ_p at the supports due to hogging of the beam is obtained by applying Mohr's theorem as,

$$\theta_p = \left(\frac{\text{Area of bending moment diagram}}{\text{Flexural rigidity}}\right) = \left(\frac{PeL}{2EI}\right)$$

where EI = flexural rigidity of the beam.



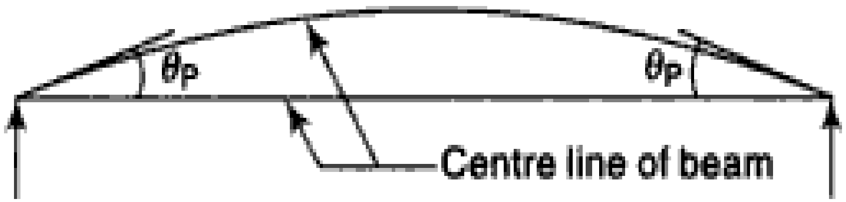


Fig. 4.21 · *Effect of Prestressing Force on Rotation of Concrete Beam*

If the beam supports a total uniformly distributed load of w_d per unit length, the rotation θ_1 at supports due to sagging of the beam is evaluated from Fig. 4.22.

$$\theta_1 = \left(\frac{\frac{1}{2} \times 2/3 \times L\, w_d L^2/8}{EI}\right) = \left(\frac{w_d L^3}{24EI}\right)$$

If the rotation due to loads is greater than that due to the prestressing force, the net rotation θ is given by,

$$\theta = (\theta_1 - \theta_p)$$

Considering Fig. 4.23,

Total elongation of the cable = $2e\theta$

Strain in the cable = $(2e\theta/L)$

Increase in stress due to loading = $\dfrac{(E_s 2e\theta)}{L}$

EXAMPLE 4.16 The cross-section of a prestressed concrete beam used over a span of 6 m is 100 mm wide and 300 mm deep. The initial stress in the tendons located at a constant eccentricity of 50 mm is 1000 N/mm^2. The sectional area of the tendons is 100 mm^2. Find the percentage increase in stress in the wires when the beam supports a live load of 4 kN/m. The density of concrete is 24 kN/m^3.
Modulus of elasticity of concrete = 36 kN/mm^2
Modulus of elasticity of steel = 210 kN/mm^2

Second moment of area $I = \left(\frac{100 \times 300^3}{12}\right) = 225 \times 10^6 \text{ mm}^4$

Prestressing force $P = (1000 \times 100) = 10^5 \text{ N} = 100 \text{ kN}$

Rotation due to prestress $\theta_p = \left(\frac{PeL}{2EI}\right) = \left(\frac{100 \times 50 \times 6 \times 10^3}{2 \times 36 \times 225 \times 10^6}\right)$

(hogging) $= 0.00185$ radians

Rotation due to loads $\theta_1 = \left(\frac{w_d L^3}{24EI}\right) = \left(\frac{0.00472 \times 6000^3}{24 \times 36 \times 225 \times 10^6}\right)$

(sagging) $= 0.00525$ radians

Net rotation = (0.00525 – 0.00185) = 0.0034 radians

Elongation of cable = 2 × 50 × 0.0034 = 0.34 mm

Increase in stress due to loading $= \left[\frac{0.34 \times 210 \times 10^3}{6000}\right] = 12\ \text{N/mm}^2$

Initial stress in cable = 1000 N/mm^2

Percentage increase in stress $= \left(\frac{12 \times 100}{1000}\right) = 1.2\%$

LOSSES OF PRESTRESS

The initial prestressing concrete undergoes a gradual reduction with time from the stages of transfer due to various causes. This is generally defined as total "Loss of Prestress". The various losses are explained below

Types of losses in prestress

Pretensioning

- Elastic deformation of concrete
- Relaxation of stress in steel
- Shrinkage of concrete
- Creep of concrete

Post-tensioning

- Relaxation of stress in steel
- Shrinkage of concrete
- Creep of concrete
- Friction
- Anchorage slip
- No loss due to elastic deformation if all wires are simultaneously tensioned. If the wires are successively tensioned, there will be loss of prestress due to elastic deformation of concrete.

Loss due to elastic deformation of the concrete

The loss of prestress due to deformation of concrete depends on the modular ratio & the average stress in concrete at the level of steel.

If f_c = prestress in concrete at the level of steel

E_s = modulus of elasticity of steel

E_c = modulus of elasticity of concrete

$$\alpha_e = \frac{E_s}{E_c} = \text{modular ratio}$$

$$\text{Strain in concrete at the level of steel} = \left(\frac{f_c}{E_c}\right)$$

$$\text{Stress in steel corresponding to this strain} = \left(\frac{f_c}{E_c}\right)E_s$$

$$\therefore \text{Loss of stress in steel} = \alpha_e f_c$$

If the initial stress in steel is known, the percentage loss of stress in steel due to the elastic deformation of concrete can be computed.

EXAMPLE 5.2 A rectangular concrete beam, 360 mm deep and 200 mm wide, is prestressed by means of fifteen 5 mm diameter wires located 65 mm from the bottom of the beam and three 5 mm wires, located 25 mm from the top of the beam. If the wires are initially tensioned to a stress of 840 N/mm^2, calculate the percentage loss of stress in steel immediately after transfer, allowing for the loss of stress due to elastic deformation of concrete only.

$$E_s = 210 \text{ kN/mm}^2$$
$$E_c = 31.5 \text{ kN/mm}^2$$

Position of the centroid of the wires from the soffit of the beam,

$$y = \left[\frac{(15 \times 65) + (3 \times 275)}{(15 + 3)}\right] = 100 \text{ mm}$$

$\therefore$ Eccentricity $e = (150 - 100) = 50$ mm

Area of concrete $A = (200 \times 300) = 6 \times 10^4 \text{ mm}^2$

$$\text{Second moment of area } I = \frac{(200 \times 300^3)}{12} = 45 \times 10^7 \text{ mm}^4$$

Prestressing force $P = (840)\,(18 \times 19.7) = 3 \times 10^5 \text{ N} = 300 \text{ kN}$

Stresses in concrete:

$$\text{At the level of top wires} = \left(\frac{300\times10^3}{6\times10^4}\right) - \left(\frac{300\times10^3\times50\times125}{45\times10^7}\right)$$

$$= 0.83\ \text{N/mm}^2$$

$$\text{At the level of bottom wires} = \left(\frac{300\times10^3}{6\times10^4}\right) + \left(\frac{300\times10^3\times50\times85}{45\times10^7}\right)$$

$$= 7.85\ \text{N/mm}^2$$

$$\text{Modular ratio } (\alpha_e) = \left(\frac{210}{31.5}\right) = 6.68$$

$$\text{Loss of stress in wires at top} = (6.68\times0.83) = 5.55\ \text{N/mm}^2$$

$$\text{Loss of stress in wires at bottom} = (6.68\times7.85) = 52.5\ \text{N/mm}^2$$

Percentage loss of stress:

$$\text{For wires at top} = \frac{5.55}{840}\times100 = 0.66\%$$

$$\text{For wires at bottom} = \frac{52.5}{840}\times100 = 6.25\%$$

Loss due to shrinkage of concrete

ε_{cs} = total residual shrinkage strain having values of 300×10^{0} for pretensioning

and $\left[\frac{200 \times 10^{-6}}{\log_{10}(t+2)}\right]$ for post-tensiomng.

where, t = age of concrete at transfer in days.

This value may be increased by 50 per cent in dry atmospheric conditions, subject to a maximum value of 300×10^{-6} units.

The loss of stress in steel due to the shrinkage of concrete is estimated as,

Loss of stress = $\varepsilon_{cs} \times E_s$.

EXAMPLE 5.6 A concrete beam is prestressed by a cable carrying an initial prestressing force of 300 kN. The cross-sectional area of the wires in the cable is 300 mm^2. Calculate the percentage loss of stress in the cable only due to shrinkage of concrete using IS: 1343 recommendations assuming the beam to be, (a) pre-tensioned and (b) post-tensioned. Assume E_s = 210 kN/mm^2 and age of concrete at transfer = 8 days.

$$\text{Initial stress in wires} = \left(\frac{300 \times 10^3}{300}\right) = 1000 \text{ N/mm}^2$$

(a) If the beam is pre-tensioned, the total residual shrinkage strain = 300×10^{-6} units.

$\therefore$ Loss of stress = $(300 \times 10^{-6})(210 \times 10^3) = 63$ N/mm^2

$$\therefore \textbf{Percentage loss of stress} = \left(\frac{63}{1000} \times 100\right) = 6.3\%$$

(b) If the beam is post-tensioned, the total residual shrinkage strain

$$= \left[\frac{200 \times 10^{-6}}{\log_{10}(8+2)}\right] = 200 \times 10^{-6} \text{ units}$$

$\therefore$ Loss of stress $= (200 \times 10^{-6})(210 \times 10^{3}) = 42 \text{ N/mm}^2$

Percentage loss of stress $= \left(\frac{42}{1000} \times 100\right) = 4.2\%$

Loss due to creep of concrete

The sustained prestress in the concrete of a prestress member results in creep of concrete which is effectively reduces the stress in high tensile steel. The loss of stress in steel due to creep of concrete can be estimated if the magnitude of ultimate creep strain or creep-coefficient is known.

1. Ultimate Creep Strain Method

If ε_{cc} = ultimate creep strain for a sustained unit stress

f_c = compressive stress in concrete at the level of steel

E_s = modulus of elasticity of steel

2. *Creep Coefficient Method*

If ϕ = creep coefficient

ε_c = creep strain

ε_e = elastic strain

α_e = modular ratio

f_c = stress in concrete

E_c = modulus of elasticity of concrete

E_s = modulus of elasticity of steel

$$\text{Creep coefficient} = \left(\frac{\text{creep strain}}{\text{elastic strain}}\right) \quad \therefore \ \phi = \left(\frac{\varepsilon_c}{\varepsilon_e}\right)$$

$$\therefore \ \varepsilon_c = \phi\varepsilon_e = \phi(f_c/E_c)$$

Hence, loss of stress in steel = $\varepsilon_c E_c \phi E_s = \phi(f_c/E_c)E_s = \phi f_c \alpha_e$

EXAMPLE 5.7 A concrete beam of rectangular section, 100 mm wide and 300 mm deep, is prestressed by five wires of 7 mm diameter located at an eccentricity of 50 mm, the initial stress in the wires being 1200 N/mm^2. Estimate the loss of stress in steel due to creep of concrete using the ultimate creep strain method and the creep coefficient method (IS: 1343–1980). Use the following data:

$E_s = 210$ kN/mm^2 $\qquad$ $I = 225 \times 10^6$ mm^4

$E_c = 35$ kN/mm^2 $\qquad$ Ultimate creep strain

$A = 3 \times 10^4$ mm^2 $\qquad$ $\varepsilon_{cc} = 41 \times 10^{-6}$ mm/mm per N/mm^2

$P = (5 \times 38.5 \times 1200) = 23 \times 10^4$ N

$\alpha_e = (E_s / E_c) = 6$

Creep coefficient (ϕ) = 1.6

Stress in concrete at the level of steel is given by

$$f_c = \left[\frac{23 \times 10^4}{3 \times 10^4} + \frac{(23 \times 10^4 \times 50)50}{225 \times 10^6}\right] = 10.2 \text{ N/mm}^2$$

1. *Ultimate Creep Strain Method*

   Loss of stress in steel $= \varepsilon_{cc} \cdot f_c \cdot E_s$

   $= (41 \times 10^{-6})(10.2)(210 \times 10^3)$

   $= 88 \text{ N/mm}^2$

2. *Creep Coefficient Method*

   Loss of stress in steel $= \phi \cdot f_c \cdot \alpha_e$

   $= (1.6 \times 10.2 \times 6)$

   $= 97.92 \text{ N/mm}^2$

Loss due to relaxation of stress in steel

Most of the codes provide for the loss of stress due to relaxation of steel as a percentage of initial stress in steel. The BIS recommends a value varying from 0 to 90 N/mm^2 for stress in wires varying from 0.5*fpu to 0.8fpu*

where Fpu=Characteristic strength of pre-stressing tendon

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Loss of stress due to friction

$$P_x = P_0 e^{-(\mu\alpha + kx)}$$

where, P_0 = prestressing force at the jacking end

μ = coefficient offriction between cable and duct

α = the cumulative angle in radians through which the tangent to the cable profile has turned between any two points under consideration

K = friction coefficient for 'wave' effect

e = 2.7183

Loss due to Anchorage slip

The magnitude of loss of stress due to the slip in anchorage is computed as follows: -

If Δ = Slip of anchorage, in mm

L = Length of the cable, in mm

A = Cross-sectional area of the cable in mm^2

E = Modulus of elasticity of steel in N/mm^2

P = Prestressing force in the cable, n N

Then,

$$\Delta = PL/AE$$

Hence, Loss of stress due to anchorage slip $P/A = E\Delta/L$

TOTAL LOSSES ALLOWED FOR A DESIGN

Type of loss	*Percentage loss of stress*	
	Pretensioning	*Post-tensioning*
Elastic shortening and bending of concrete	4	1
Creep of concrete	6	5
Shrinkage of concrete	7	6
Creep in steel	8	8
Total	25	20

EXAMPLE 5.15 A prestressed concrete pile, 250 mm square, contains 60 pre-tensioned wires, each of 2 mm diameter, uniformly distributed over the section. The wires are initially tensioned on the prestressing bed with a total force of 300 kN. Calculate the final stress in concrete and the percentage loss of stress is steel after all losses, given the following data:

$$E_s = 210 \text{ kN/mm}^2$$
$$E_c = 32 \text{ kN/mm}^2$$

Shortening due to creep = 30×10^{-6} mm/mm per N/mm^2 of stress

Total shrinkage = 200×10^{-6} per unit length

Relaxation of steel stress = 5 per cent of initial stress

Prestressing force, $P = 300$ kN

$$\text{Average initial stress in concrete} = \left(\frac{300 \times 10^3}{250 \times 250}\right) = 4.8 \text{ N/mm}^2$$

$$\text{Modular ratio, } \alpha = \left(\frac{E_s}{E_c}\right) = 6.58$$

$$\text{Initial stress in steel wires} = \left(\frac{300 \times 10^3}{188.4}\right) = 1590 \text{ N/mm}^2$$

Losses of stress

1. Elastic deformation = (6.58×4.8) = $31.5\ N/mm^2$
2. Creep of concrete = $(30 \times 10^{-6})\ 4.8 \times 210 \times 10^3$ = $30\ N/mm^2$
3. Shrinkage of concrete = $(200 \times 10^{-6})\ 210 \times 10^3$ = $42\ N/mm^2$
4. Relaxation of steel stress = $(5/100 \times 1590)$ = $79.5\ N/mm^2$

Total loss = $183\ N/mm^2$

Effective prestress = $(1590 - 183) = 1407\ N/mm^2$

$$\text{Final stress in concrete} = \left[\frac{(1407)(1884)}{250 \times 250}\right] = 4.26\ N/mm^2$$

$$\text{Percentage loss of stress in steel} = \left(\frac{183}{1590} \times 100\right) = 11.6\%$$

DEFLECTIONS OF PSC MEMBERS

Factors influencing deflection:

1. Imposed load & self load
2. Magnitude of prestressing force
3. Cable profile
4. Second moment of area of cross-section
5. Modulus of elasticity of concrete
6. Shrinkage, creep & relaxation of steel stress
7. Span of the member
8. Fixity condition

EFFECT OF TENDON PROFILE ON DEFLECTION

1. Straight tendons

If upward deflections are considered as negative and,

P = effective prestressing force

e = eccentricity

L = length of the beam

$$a = -\frac{(PeL)(L/4)}{EI} = -\frac{PeL^2}{8EI}$$

2. Trapezoidal tendons A draped tendon with a trapezoidal profile is shown in Fig. 6.3. Considering the B.M.D., the deflection at the centre of the beam is obtained by taking the moment of area of the B.M.D. over one-half of the span. Thus,

$$a = -\frac{Pe}{EI}[l_2(l_1 + l_2/2) + (l_1/2)(2/3l_1)] = -\frac{Pe}{6EI}[2l_1^2 + 6l_1 l_2 + 3l_2^2]$$

3. Parabolic tendons (central anchors) The deflection of a beam with parabolic tendons (Fig. 6.4) having an eccentricity e at, the centre and zero at the supports is given by,

$$a = -\frac{Pe}{EI}\left[\frac{2}{3}\cdot\frac{L}{2}\cdot\frac{5}{8}\cdot\frac{L}{2}\right] = -\left(\frac{5PeL^2}{48EI}\right)$$

4. Parabolic tendons (eccentric anchors)

$$a = \frac{PL^2}{48EI}(-5e_1 + e_2)$$

5. Sloping tendons (eccentric anchors)

$$a = \left[-\frac{PL^2}{12EI}(e_1 + e_2)\right] + \left[\frac{Pe_2L^2}{8EI}\right] = \frac{PL^2}{24EI}(-2e_1 + e_2)$$

Deflections due to Self-Weight and Imposed Loads

At the time of transfer of prestress, the beam hogs up due to the effect of prestressing. At this stage, the self-weight of the beam induces downward deflections, which further increase due to the effect of imposed loads on the beam.

If g = self-weight of the beam/m

q = imposed load/m (uniformly distributed),

the downward deflection is computed as,

$$a = \frac{5(g+q)L^4}{384EI}$$

EXAMPLE 6.2 A concrete beam with a cross-sectional area of 32×10^3 mm^2 and radius of gyration of 72 mm is prestressed by a parabolic cable carrying an effective stress of 1000 N/mm^2. The span of the beam is 8 m. The cable, composed of 6 wires of 7 mm diameter, has an eccentricity of 50 mm at the centre and zero at the supports. Neglecting all losses, find the central deflection of the beam as follows:

(a) self-weight + prestress, and
(b) self-weight + prestress + live load of 2 kN/m.

Assume $E = 38$ kN/mm^2 and $D_c = 24$ kN/m^3

$A = 32 \times 10^3$ mm^2, $i = 72$ mm, $L = 8000$ mm, $e = 50$ mm

$I = A\,i^2 = (32 \times 10^3 \times 72^2) = 166 \times 10^6$ mm^4

$P = (6 \times 38.5 \times 1000) = 231000$ N $= 231$ kN

$$g = \left(\frac{32 \times 10^3}{10^6} \times 24\right) = 0.77 \text{ kN/m} = 0.00077 \text{ kN/mm}$$

Downward deflection due to self-weight $= \left(\dfrac{5\,gL^4}{384EI}\right) = \left(\dfrac{5 \times 0.00077 \times 8000^4}{384 \times 38 \times 166 \times 10^6}\right)$

$= 6.5$ mm

Upward deflection due to prestressing force $= \left(\frac{5PeL^2}{48EI}\right) = \left(\frac{5\times 231\times 50\times 8000^2}{48\times 38\times 166\times 10^6}\right)$

$= 12.2$ mm

Downward deflection due to live load $= \left(\frac{6.5}{0.77}\times 2\right) = 16.9$ mm

(a) Deflection due to (self-weight + prestress) = (12.2 – 6.5) = 5.7 mm (upwards)

(b) Deflection due to (self-weight + prestress + live load) = (6.5 – 12.2 + 16.9) = 11.2 mm (downwards)

Prediction of long time deflection

the total long time deflection after time t is obtained from the expression,

$$a_f = a_{i1}(1+\phi) - a_{ip}\left[\left(1-\frac{L_p}{P_i}\right)+\left(1-\frac{L_p}{2P_i}\right)\phi\right]$$

$$a_f = \left[a_{i1} - a_{ip} \times \frac{P_t}{P_i}\right](1+\phi)$$

P_i = initial prestress

P_t = prestress after a time, t

Loss of prestressing force due to relaxation, shrinkage and creep, $L_p = (P_i - P_t)$

e = eccentricity of the prestressing force at the section

EI = flexural rigidity

Example 6.4 A concrete beam having a rectangular section 100 mm wide and 300 mm deep is prestressed by a parabolic cable carrying an initial force of 240 kN. The cable has an eccentricity of 50 mm at the centre of span and is concentric at the supports. If the span of the beam is 10 m and the live load is 2 kN/m, estimate the short time deflection at the centre of span.
Assuming $E = 38$ kN/mm^2 and creep coefficient $\phi = 2.0$, loss of prestress = 20 per cent of the initial stress after 6 months. Estimate the long time deflection at the centre of span at this stage, assuming that the dead and live loads are simultaneously applied after the release of prestress.

$P_i = 240$ kN	
$I = 225 \times 10^6$ mm^4	Dead load = 0.72 kN/m
$e = 50$ mm	Live load = 2 kN/m
$E = 38$ kN/mm^2	Loss of prestress = 20 per cent
$\phi = 2.0$	$= 0.2\, P_i$

(a) *Short time or instantaneous deflection*

$$\text{Deflection due to prestress} = \left(\frac{5P_i eL^2}{48EI}\right) = \left(\frac{5 \times 240 \times 50 \times (10 \times 1000)^2}{48 \times 38 \times 225 \times 10^6}\right)$$

$$= 14.7 \text{ mm (upward)}$$

Deflection due to self-weight and live loads = $\left[\dfrac{5(g+q)L^4}{384EI}\right]$

$$= \left[\frac{5\times(0.00072+0.002)(10\times1000)^4}{384\times38\times225\times10^6}\right]$$

$= 41.5$ mm (downward)

$\therefore$ Net deflection $= (41.5 - 14.7) = 26.8$ mm (downward)

(b) Long time deflection

Initial deflection due to transverse loads $= 41.5$ mm

Initial deflection due to prestress only $= 14.7$ mm

Hence, the final deflection is computed as,

$$a_f = 41.5(1+2) - 14.7\left[1 - \frac{0.2P_i}{P_i} + \left(1 - \frac{0.2P_i}{2P_i}\right)2\right]$$

$= (124.5 - 38) = 86.5$ mm (downward)