

DESIGN CONCEPT

Flexural strength of prestressed concrete member

Types of flexural failure

1. Fracture of steel:-

A minimum longitudinal reinforcement of 0.2% of the total concrete area shall be provided in all the cases except in the case of prestressed units of small sections. This reinforcement may be further reduced to 0.15% in the case of HYSD bars. The percentage of steel provided, both tensioned & un-tensioned taken together should be sufficient so that when the concrete in precompressed tensile zone cracks, the steel is in position to take up the additional tensile stress, transferred on to it by the cracking of the adjacent fiber of concrete & a sudden failure is avoided.

2. Failure of over reinforced section:-

When the effective reinforcement index ρ_{eff} , which is expressed in terms of the percentage of reinforcement, the compressive strength of the concrete and the tensile strength of the steel, exceeds a certain range of values, the section is said to be over reinforced. Generally, over-reinforced members fail by the sudden crushing of concrete, the failure being characterized by small deflection and narrow cracks. The area of steel being comparatively large, the stresses developed in steel at failure of the member may not reach the tensile strength & in many cases it may well be within the proof stress of the tendon.

3. Failure of under reinforced section: -

If the cross-section is provided with a steel greater than the minimum prescribed above, the failure is characterized by an excessive elongation of steel followed by crushing of concrete. This type of behaviors is generally desirable since there is considerable warning before the impending failure. As such, it is common practice to design the under-reinforced sections, which become more important in case of statically indeterminate structure.

Design of Prestressed Concrete Member

Indian code (as per IS 1343 1980)

Assumption

1. The plane sections normal to the axis remain plane after bending.
2. The maximum strain in concrete at outermost compression fibre is taken as 0.35% in bending regardless of the strength of concrete.
3. The relationship between stress & strain distribution in concrete is assumed to be parabolic. The maximum compressive stress is equal to $0.446 f_{ck}$
Where, f_{ck} = *Characteristic strength of the concrete*
4. The tensile strength of concrete is ignored.
5. The steel & concrete are bonded completely.
6. The stresses in bonded prestressing tendons are derived from the respective stress-strain curve for the particular steel.

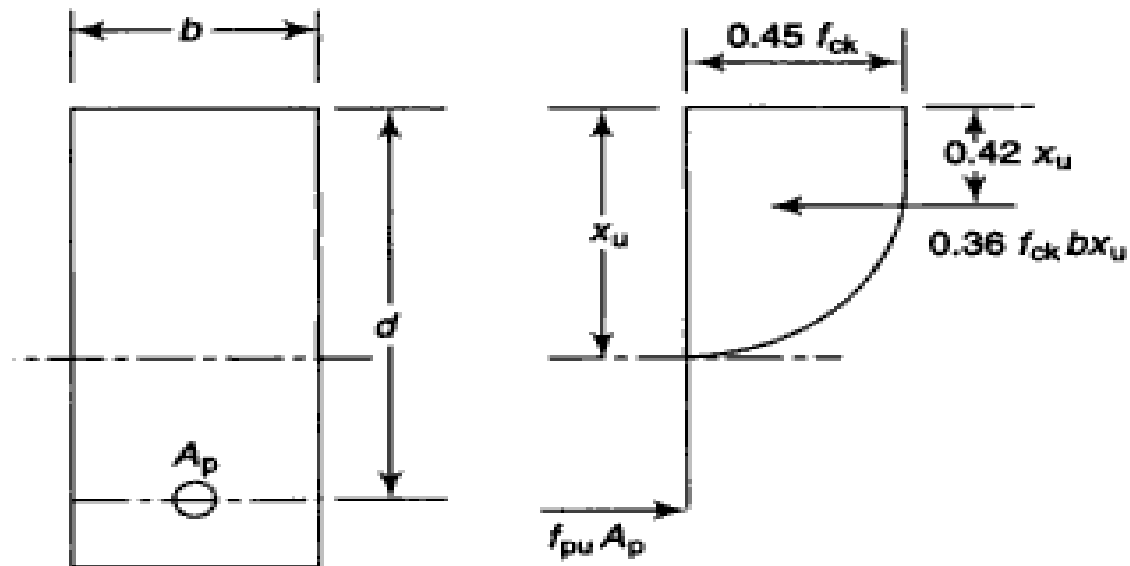


Fig. 7.7 *Moment of Resistance of Rectangular Sections (IS:1343–1980)*

The moment of resistance is obtained from the equation,

$$M_u = f_{pu} A_p (d - 0.42 x_u)$$

where

M_u = ultimate moment of resistance of the section

f_{pu} = tensile stress developed in tendons at the failure stage of the beam

f_p = characteristic tensile strength of the prestressing steel

f_{pe} = effective prestress in tendons after losses

A_p = area of prestressing tendons

d = effective depth

x_u = neutral-axis depth

The value of f_{pu} depends upon the effective reinforcement ratio

$$\left(\frac{A_p f_p}{b d f_{ck}} \right)$$

Moment of resistance of flanged section

The ultimate moment of resistance of flanged sections in which the neutral axis falls outside the flange is computed by combining the moment of resistance of web & flange portion & considering the stress block is shown below

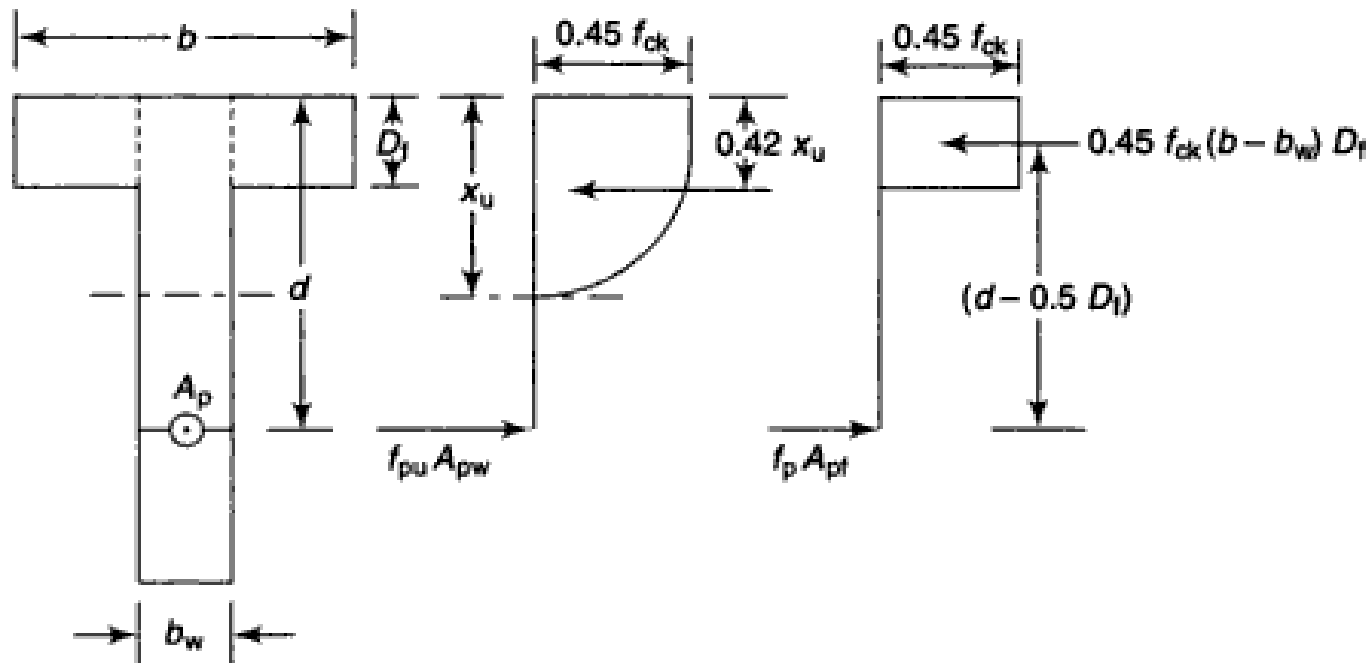


Fig. 7.8 Moment of Resistance of Flanged Sections ($x_u > D_f$) (IS: 1343-1980)

If A_{pw} = area of prestressing steel for web
 A_{pf} = area of prestressing steel for flange
 D_f = thickness of flange

Then, $A_p = (A_{pw} + A_{pf})$

But, $A_{pf} = 0.45f_{ck}(b - b_w) \left(\frac{D_f}{f_p} \right)$

After evaluating A_{pf} , the value of A_{pw} is obtained as

$$A_{pw} = (A_p - A_{pf})$$

For the effective reinforcement ratio of $(A_{pw}f_p/b_wdf_{ck})$, the corresponding values of $(f_{pu}/0.87f_p)$ and (x_u/d) are obtained from Table 7.1. The ultimate moment of resistance of the flanged section is obtained from the expression,

$$M_u = f_{pu} \cdot A_{pw} (d - 0.42 x_u) + 0.45f_{ck} (b - b_w)D_f(d - 0.5 D_f)$$

EXAMPLE 7.3 A pretensioned prestressed concrete beam having a rectangular section, 150 mm wide and 350 mm deep, has an effective cover of 50 mm. If $f_{ck} = 40 \text{ N/mm}^2$, $f_p = 1600 \text{ N/mm}^2$, and the area of prestressing steel $A_p = 461 \text{ mm}^2$, calculate the ultimate flexural strength of the section using 18:1343 code provisions.

Given data: $f_{ck} = 40 \text{ N/mm}^2$ $b = 150 \text{ mm}$
 $f_p = 1600 \text{ N/mm}^2$ $d = 300 \text{ mm}$
 $A_p = 461 \text{ mm}^2$

The effective reinforcement ratio is given by

$$\left(\frac{f_p A_p}{f_{ck} b d} \right) = \left(\frac{1600 \times 461}{40 \times 150 \times 300} \right) = 0.40$$

From Table 7.1, the corresponding values of

$$\left(\frac{f_{pu}}{0.87 f_p} \right) = 0.9 \text{ and } \left(\frac{x_u}{d} \right) = 0.783$$

$$\therefore f_{pu} = (0.87 \times 0.9 \times 1600) = 1253 \text{ N/mm}^2$$

$$x_u = (0.783 \times 300) = 234.9 \text{ mm}$$

$$\begin{aligned} M_u &= f_{pu} A_p (d - 0.42 x_u) \\ &= 1253 \times 461 (300 - 0.42 \times 234.9) \\ &= 116 \times 10^6 \text{ N mm} = 116 \text{ kN m} \end{aligned}$$

EXAMPLE 7.5 A pretensioned, T-section has a flange 1200 mm wide and 150 mm thick. The width and depth of the rib are 300 and 1500 mm respectively. The high-tensile steel has an area of 4700 mm² and is located at an effective depth of 1600 mm. If the characteristic cube strength of the concrete and the tensile strength of steel are 40 and 1600 N/mm² respectively, calculate the flexural strength of the T-section

Given data:

$$A_p = 4700 \text{ mm}^2 \qquad d = 1600 \text{ mm}$$

$$f_{ck} = 40 \text{ N/mm}^2 \qquad D_f = 150 \text{ mm}$$

$$b = 1200 \text{ mm}$$

$$b_w = 300 \text{ mm}$$

$$A_p = (A_{pw} + A_{pf})$$

$$A_{pf} = 0.45 f_{ck} (b - b_w) \left(\frac{D_f}{f_p} \right)$$

$$= 0.45 \times 40 (1200 - 300) \left(\frac{150}{1600} \right) = 1518 \text{ mm}^2$$

∴

$$A_{pw} = (4700 - 1518) = 3182 \text{ mm}^2$$

Also
$$\left(\frac{A_{pw} f_p}{b_w d f_{ck}} \right) = \left(\frac{3182 \times 1600}{300 \times 1600 \times 40} \right) = 0.265$$

From Table 7.1 or (Table 11 of IS: 1343-1980),

$$\left(\frac{f_{pu}}{0.87 f_p} \right) = 1.00 \quad \therefore \quad f_{pu} = (0.87 \times 1600) = 1392 \text{ N/mm}^2$$

$$\left(\frac{x_u}{d} \right) = 0.56 \quad \therefore \quad x_u = (0.56 \times 1600) = 896 \text{ mm}$$

$$\begin{aligned} M_u &= f_{pu} A_{pw} (d - 0.42 x_u) + 0.45 f_{ck} (b - b_w) D_f (d - 0.5 D_f) \\ &= 1392 \times 3182 (1600 - 0.42 \times 896) + 0.45 \times 40 \times 900 \times 150 (1600 - 75) \\ &= (5420 \times 10^6) + (3705 \times 10^6) \\ &= 9125 \times 10^6 \text{ N mm} = 9125 \text{ kN m} \end{aligned}$$

7.3.2 British Code Provisions

The British code BS: 8110–1985 provides that for prestressed concrete members, the stress distribution in concrete at failure may be assumed to be rectangular with an average stress value of $0.45 f_{cu}$ and the depth of the stress block is assumed to be equal to 0.9 times the depth of the compression zone as shown in Fig. 7.9. The ultimate moment of resistance of a beam containing bonded or unbonded tendons, all of which are located in the tension zone, may be obtained from the equation:

$$M_u = A_{pb} A_{ps} (d - d_n)$$

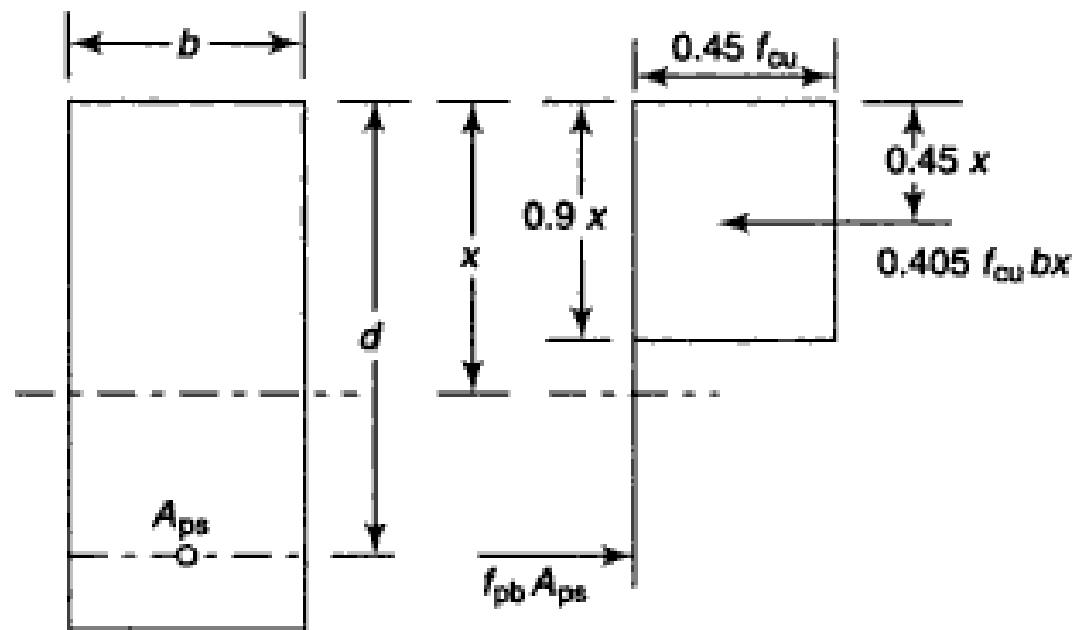


Fig. 7.9 Moment of Resistance of Rectangular Sections (BS : 8110–1985)

For a rectangular or flanged beam in which the flange thickness is not less than $0.9x$, d_n may be taken as $0.45x$.

For bonded tendons, the values of f_{pb} and x may be obtained from Table 7.3.

Table 7.3 *Conditions at the Ultimate Limit State for Rectangular Beams with Pretensioned Tendons or Post-tensioned Tendons having Effective Bond (BS: 8110-1985)*

$\left(\frac{f_{pu} A_{ps}}{f_{cu} bd} \right)$	Design stress in tendons as a proportion of the design strength, $(f_{pb}/0.87f_{pu})$			Ratio of the depth of neutral axis to that of the centroid of the tendons in the tension zone (x/d)		
	$(f_{pe}/f_{pu}) =$			$(f_{pe}/f_{pu}) =$		
	0.6	0.5	0.4	0.6	0.5	0.4
0.05	1.0	1.0	1.0	0.11	0.11	0.11
0.10	1.0	1.0	1.0	0.22	0.22	0.22
0.15	0.99	0.97	0.95	0.32	0.32	0.31
0.20	0.92	0.90	0.88	0.40	0.39	0.38
0.25	0.88	0.86	0.84	0.48	0.47	0.46
0.30	0.85	0.83	0.80	0.55	0.54	0.52
0.35	0.83	0.80	0.76	0.63	0.60	0.58
0.40	0.81	0.77	0.72	0.70	0.67	0.62
0.45	0.79	0.74	0.68	0.77	0.72	0.66
0.50	0.77	0.71	0.64	0.83	0.77	0.69

For unbonded tendons, values of f_{pb} and x may be obtained from the following equations:

$$f_{pb} = f_{pe} + \frac{7000}{(L/d)} \left[1 - 1.7 \left(\frac{f_{pu} A_{ps}}{f_{cu} bd} \right) \right]$$

and

$$x = 2.47 \left[\left(\frac{f_{pu} A_{ps}}{f_{cu} bd} \right) \left(\frac{f_{pb}}{f_{pu}} \right) d \right]$$

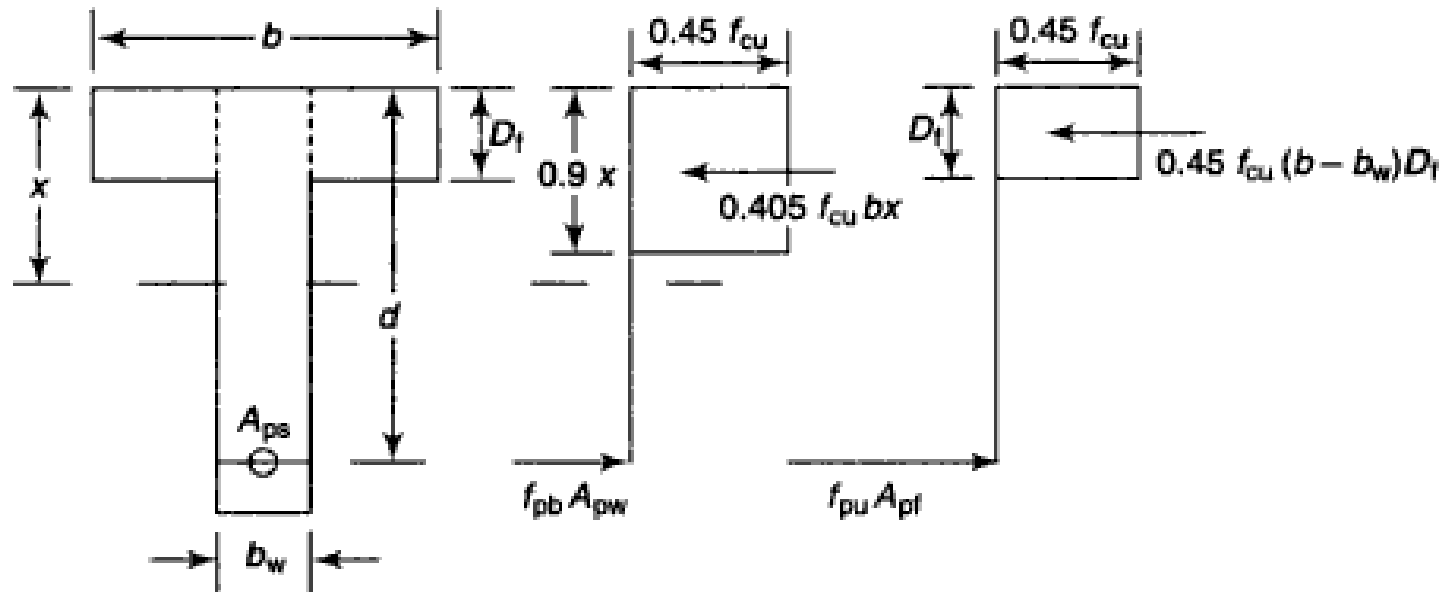


Fig. 7.10 Moment of Resistance of Flanged Sections ($x > D_f$) (BS: 8110-1985)

$$M_u = f_{pb} A_{pw} (d - 0.45x) + 0.54 f_{cu} (b - b_w) D_f (d - 0.5D_f)$$

where,

$$A_{pw} = (A_{ps} - A_{pf})$$

$$A_{pf} = 0.45 f_{cu} (b - b_w) \left(\frac{D_f}{f_{pu}} \right)$$

EXAMPLE 7.9 A prestressed concrete beam of effective span 16 m is of rectangular section 400 mm wide by 1200 mm deep. The tendons consist of 3300 mm² of strands of characteristic strength 1700 N/mm², with an effective prestress of 910 N/mm². The strands are located 870 mm from the top face of the beam. If $f_{cu} = 60$ N/mm², estimate the flexural strength of the section as per British Code provisions for the following cases:

(a) Bonded tendons

(b) Unbonded tendons

Given data:

$$f_{pu} = 1700 \text{ N/mm}^2$$

$$L = 16 \text{ m}$$

$$f_{cu} = 60 \text{ N/mm}^2$$

$$b = 400 \text{ m}$$

$$A_{ps} = 3300 \text{ mm}^2$$

$$d = 870 \text{ m}$$

$$f_{pe} = 910 \text{ N/mm}^2$$

$$(L/d) = 18.39 \text{ m}$$

For bonded tendons, the ratio

$$\left(\frac{f_{pu} A_{ps}}{f_{cu} bd} \right) = \left(\frac{1700 \times 3300}{60 \times 400 \times 870} \right) = 0.27$$

and

$$\left(\frac{f_{pe}}{f_{pu}} \right) = \left(\frac{910}{1700} \right) = 0.54$$

From Table 7.3, by interpolation,

$$\left(\frac{f_{pb}}{0.87 f_{pu}} \right) = 0.86 \quad \text{and} \quad \left(\frac{x}{d} \right) = 0.50$$

Hence,

$$f_{pb} = (0.86 \times 0.87 \times 1700) = 1272 \text{ N/mm}^2$$

and

$$x = (0.50 \times 870) = 435 \text{ mm}$$

\therefore

$$\begin{aligned} M_u &= f_{pb} A_{ps} (d - 0.45x) \\ &= 1272 \times 3300(870 - 0.45 \times 435) \text{ N mm} = 2830 \text{ kN m} \end{aligned}$$

For unbonded tendons,

$$f_{pb} = f_{pe} + \frac{7000}{(L/d)} \left[1 - 1.7 \left(\frac{f_{pu} A_{ps}}{f_{cu} bd} \right) \right]$$
$$= 910 + \frac{7000}{18.39} [1 - 1.7 (0.27)] = 1116 \text{ N/mm}^2$$

and,

$$x = 2.47 \left[\left(\frac{f_{pu} A_{ps}}{f_{cu} bd} \right) \left(\frac{f_{pb}}{f_{pu}} \right) d \right]$$
$$= 2.47 \left[0.27 \left(\frac{1116}{1700} \right) 870 \right] = 380.8 \text{ mm}$$

Hence,

$$M_u = f_{pb} A_{ps} (d - 0.45 x)$$
$$= 1116 \times 3300 (870 - 0.45 \times 380.8) \text{ N mm} = 2573 \text{ kN m}$$

EXAMPLE 7.10 A post-tensioned, prestressed concrete girder is of T-section with an effective flange width and depth of 1500 mm and 250 mm respectively. Thickness of the web is 200 mm. The area of prestressing steel is 5000 mm², located at an effective depth of 1600 mm. Given $f_{pu} = 1600 \text{ N/mm}^2$, $f_{cu} = 40 \text{ N/mm}^2$ and $f_{pe} = 960 \text{ N/mm}^2$, estimate the ultimate moment of resistance of the T-section. Assume the effective span of the girder as 32 m.

Given data:

$$A_{ps} = 5000 \text{ mm}^2$$

$$f_{cu} = 40 \text{ N/mm}^2$$

$$f_{pu} = 1600 \text{ N/mm}^2$$

$$f_{pe} = 960 \text{ N/mm}^2$$

$$b = 1500 \text{ mm}$$

$$b_w = 200 \text{ mm}$$

$$D_f = 250 \text{ mm}$$

$$d = 1600 \text{ mm}$$

$$L = 32 \text{ m}$$

$$\left(\frac{L}{d}\right) = \left(\frac{32}{1.6}\right) = 20$$

$$\left(\frac{f_{pe}}{f_{pu}}\right) = 0.6$$

Thus,

$$A_{ps} = A_{pw} + A_{pf}$$

$$A_{pf} = 0.45 f_{ck}(b - b_w) \left(\frac{D_f}{f_{pu}} \right)$$

$$= 0.45 \times 40(1500 - 200) \left(\frac{250}{1600} \right) = 3656 \text{ mm}^2$$

$$\therefore A_{pw} = (5000 - 3656) = 1344 \text{ mm}^2$$

and
$$\left(\frac{f_{pu} A_{pw}}{f_{cu} b_w d} \right) = \left(\frac{1600 \times 1344}{40 \times 200 \times 1600} \right) = 0.168$$

From Table 7.3, by interpolation,

$$\left(\frac{f_{pb}}{0.87 f_{pu}} \right) = 0.95$$

$$\therefore f_{pb} = (0.95 \times 0.87 \times 1600) = 1322 \text{ N/mm}^2$$

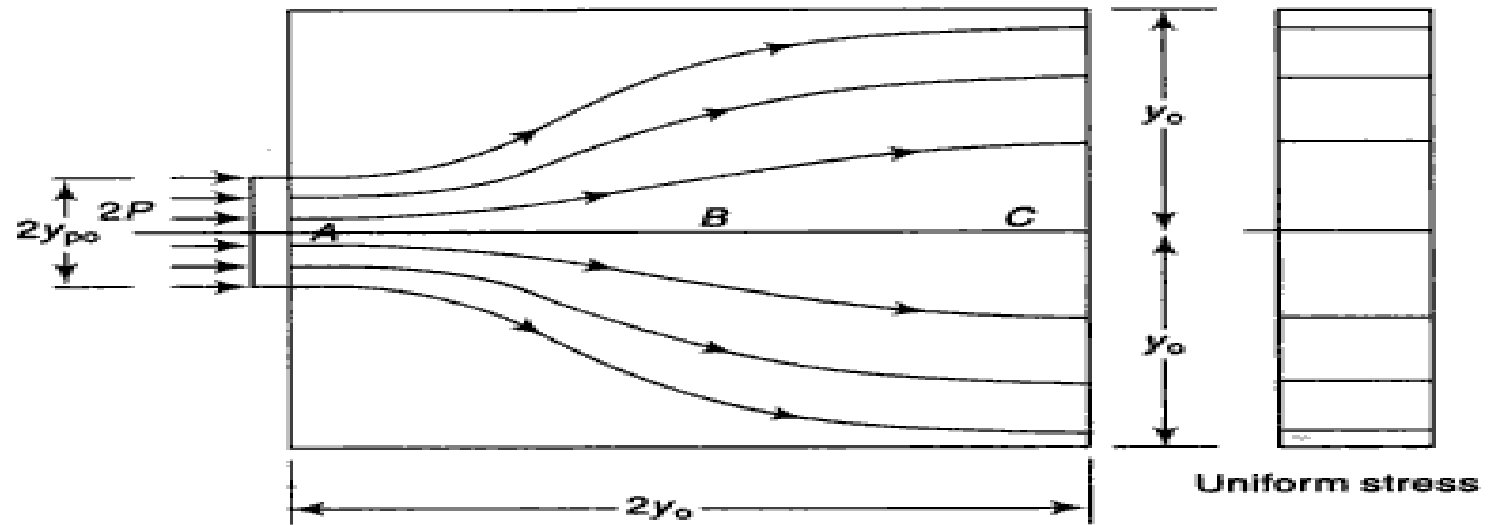
$$\left(\frac{x}{d}\right) = 0.35$$

$$\therefore x = (0.35 \times 1600) = 560 \text{ mm}$$

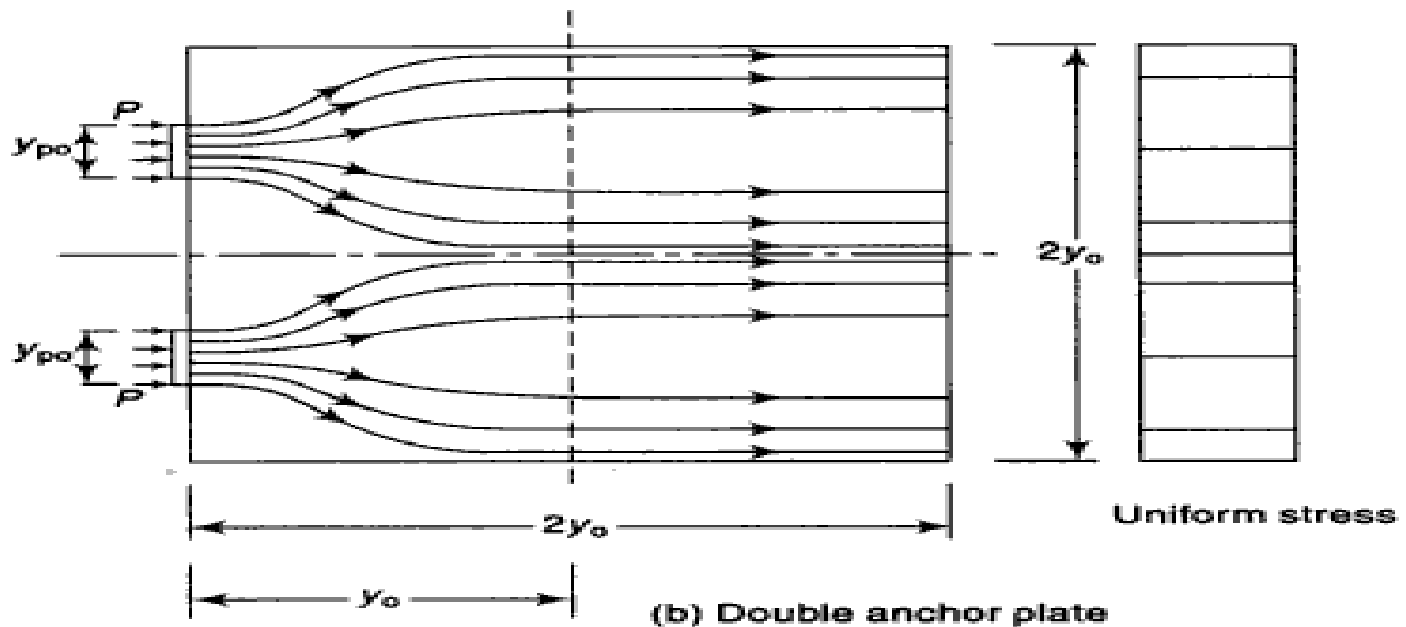
$$\begin{aligned}\therefore M_u &= f_{pb} A_{pw} (d - 0.45x) + 0.45 f_{cu} (b - b_w) D_f (d - 0.45 D_f) \\ &= 1322 \times 1344 (1600 - 0.45 \times 560) + 0.45 \times 40 (1500 - 200) \\ &\quad \times 250 (1600 - 0.5 \times 250) \\ &= 11,023 \times 10^6 \text{ N mm} = 11,023 \text{ kN m}\end{aligned}$$

Stress distribution in end blocks

- ▶ In the anchorage zone or end block of a post-tensioned concrete member the state of stress distribution is complex and 3D in nature.
- ▶ In most post tensioned member, the pre stressing wires are introduced in cable holes or ducts, pre formed in the members and then the stresses anchored at the end forces
- ▶ **Anchorage zone**
The zone between the end of the beam and the section where only longitudinal stress exists is generally referred to as the anchorage zone or end block



(a) Single anchor plate



(b) Double anchor plate

Fig. 10.1 *Transmission of Forces in End Block*

The idealised stress distribution in an end block with the compressive and tensile stress paths is shown in Fig. 10.2 (a). The effect of transverse tensile stress is to develop a zone of bursting tension in a direction perpendicular to the anchorage force, resulting in horizontal cracking as shown in Fig. 10.2 (b). Since concrete is weak in tension, suitable reinforcements are generally provided in the transverse direction to resist the bursting tension.

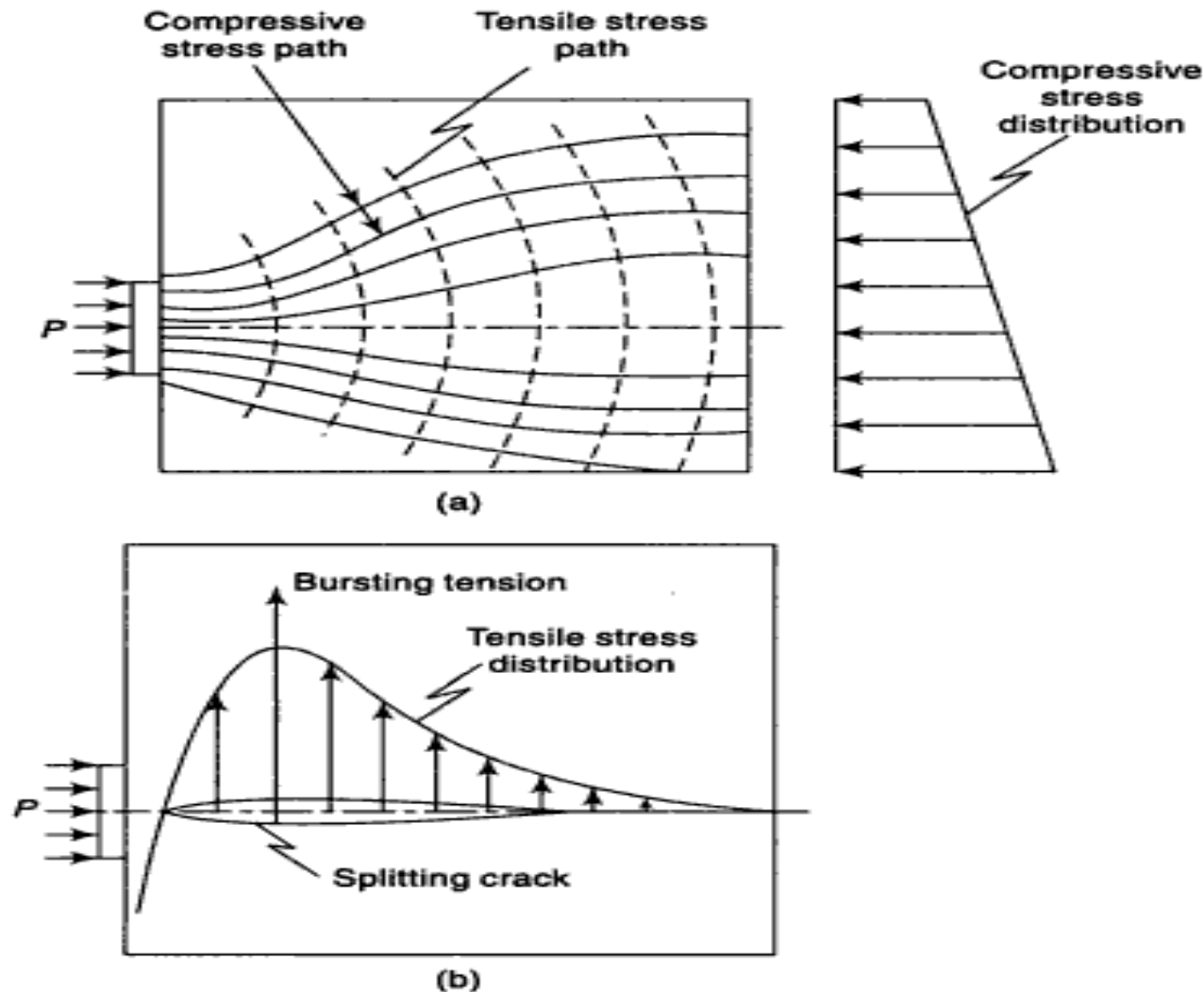


Fig. 10.2 End Blocks of Post-Tensioned Beams (a) Idealized Stress Paths (b) Bursting Tension and Splitting Cracks

INVESTIGATION OF ANCHORAGE ZONE STRESSES

Magnel's method

In this method the end block is considered as a deep beam subjected to concentrated loads due to anchorage on one side and to normal and tangential distribution from the linear direct stress and shear stress distribution from other side.

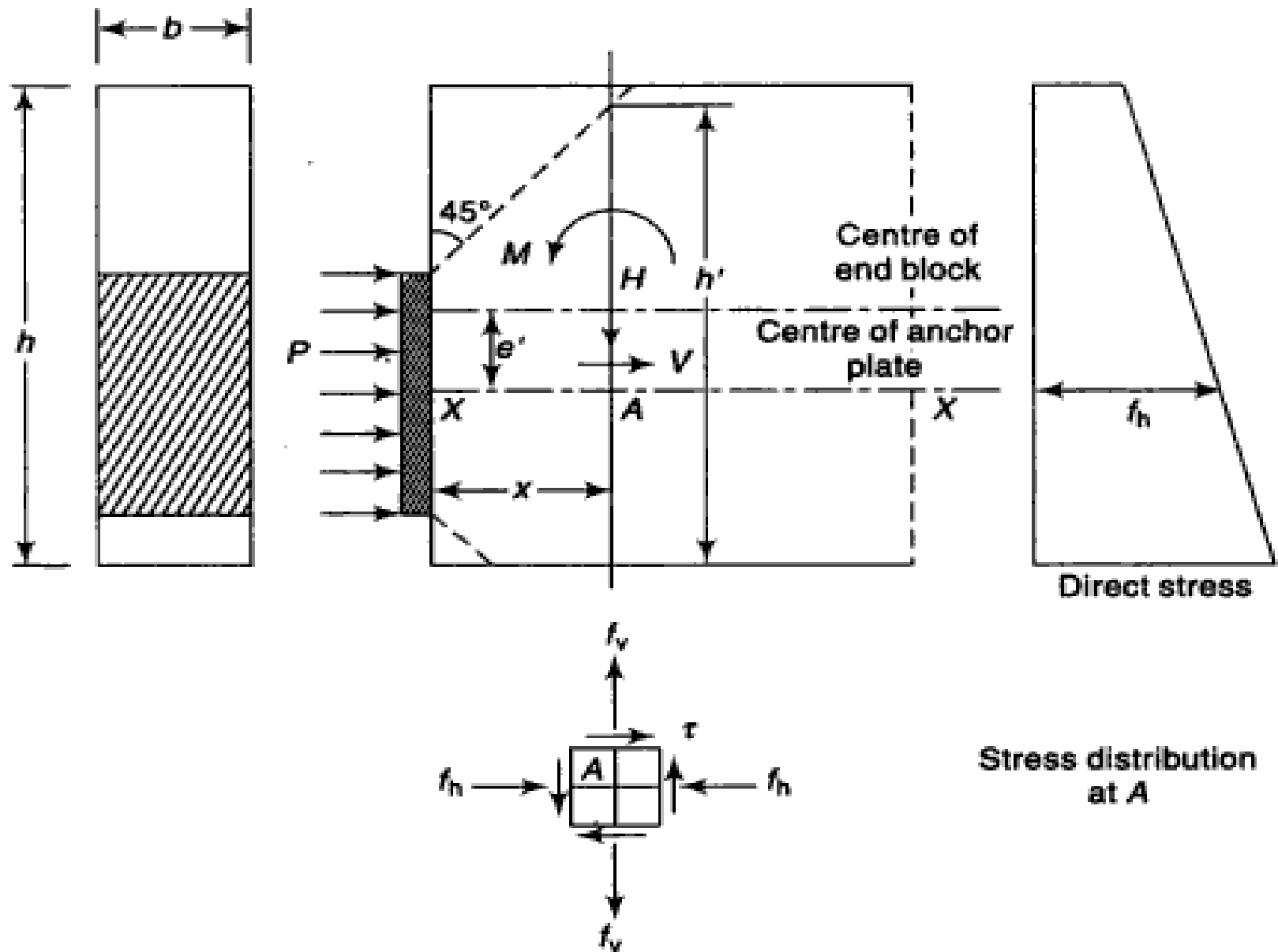


Fig. 10.4 Forces Acting on the End Block

M = bending moment

H = direct force (vertical) (directions shown in the figure are +ve)

V = shear force (horizontal)

f_v = vertical stress

f_h = direct stress (at point A shown in the figure)

τ = shear stress

The stress distribution across the section can be approximated by the following equations:

$$f_v = K_1 \left(\frac{M}{bh^2} \right) + K_2 \left(\frac{H}{bh} \right)$$

$$\tau = K_3 \left(\frac{V}{bh} \right)$$

$$f_h = \frac{P}{bh} \left(1 + 12 \frac{e'^2}{h'^2} \right)$$

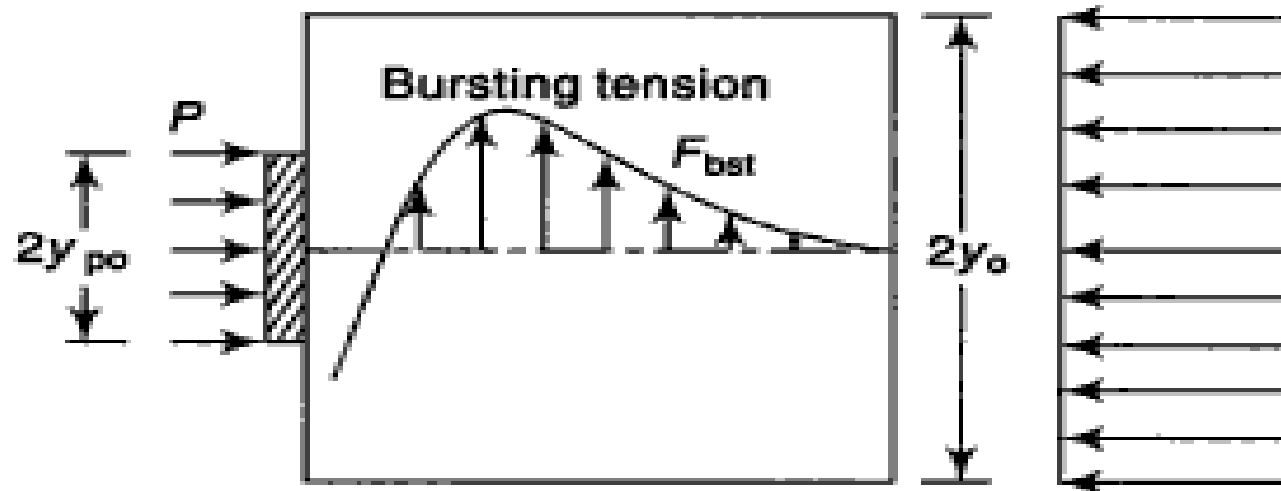
The principal stresses acting at the point are computed by the general equations:

$$f_{\max} \text{ or } f_{\min} = \left(\frac{f_v + f_h}{2} \right) \pm \frac{1}{2} \sqrt{(f_h - f_v)^2 + 4\tau^2}$$

$$\tan 2\theta = \left(\frac{2\tau}{f_v - f_h} \right)$$

Guyon's method

- ▶ Guyon method developed for computation of bursting tension in end block which are based on this investigation concerning the distribution of stresses in end block subjected to concentrated loads.



Stress – distribution

Fig. 10.8 *Evenly Distributed Force System (Guyon)*

bursting tension is expressed as,

$$F_{bst} = 0.3P[1 - (y_{po}/y_o)^{0.58}]$$

where

P = anchorage force

y_{po}/y_o = distribution ratio

where

$2y_{po}$ = depth of the anchorage plate

$2y_o$ = depth of the equivalent prism

Zielinski and rowe's method

$$\text{Tensile stress, } f_{v(\max)} = f_c \left[0.98 - 0.825 \left(\frac{y_{po}}{y_o} \right) \right]$$

$$\text{valid for ratio of } (y_{po}/y_o) = 0.3 \text{ to } 0.7$$

$$\text{and Bursting Tension, } F_{bst} = P_k \left[0.48 - 0.4 \left(\frac{y_{po}}{y_o} \right) \right]$$

$$F_{bst}(\text{corrected}) = F_{bst} \left[1 - \left(\frac{f_t}{f_{v(\max)}} \right)^2 \right]$$

$2y_o$ = side of the surrounding prism (similar to the equivalent prism of Guyon's method)

$2y_{po}$ = side of loaded or punching area

y_{po}/y_o = ratio of sides of loaded to bearing area of the prism

f_v = transverse tensile stress

EXAMPLE 10.3 Using Guyon's method, compute the position and magnitude of maximum tensile stress and bursting tension for the end block with concentric anchor. The end block of a prestressed concrete beam, rectangular in section, is 100 mm wide and 200 mm deep. The prestressing force of 100 kN is transmitted to concrete by a distribution plate, 100 mm wide and 50 mm deep, concentrically located at the ends.

$$P = 100 \text{ kN}$$

$$2y_{p0} = 50 \text{ mm}$$

$$2y_o = 200 \text{ mm}$$

$$\therefore \text{Distribution ratio} \quad y_{p0}/y_o = 0.25$$

From Table 10.2,

$$\text{Position of zero stress from the end face} = 0.15(2y_o) = 30 \text{ mm}$$

$$\text{Position of maximum stress} = 0.33(2y_o) = 66 \text{ mm}$$

$$\begin{aligned}\text{Maximum tensile stress} &= 0.345(P/A) = 0.345 \left(\frac{100 \times 10^3}{200 \times 100} \right) \\ &= 1.725 \text{ N/mm}^2\end{aligned}$$

$$\begin{aligned}\text{Bursting tension, } F_{bst} &= 0.3P[1 - (y_{po}/y_o)^{0.58}] \\ &= 0.3 \times 100 \times 10^3 [1 - (0.25)^{0.58}] = 16575 \text{ N}\end{aligned}$$

If the yield stress in mild steel = 260 N/mm^2 , then

$$\text{Area of steel required} = \left[\frac{16575}{(0.87 \times 260)} \right] = 73 \text{ mm}^2$$

EXAMPLE 10.4 For the rectangular end block with an eccentric anchor force of 100 kN, as detailed in Example 10.2, compute the maximum tensile stress and the total splitting tension using Guyon's method.

The end block of a prestressed concrete beam, 100 mm wide and 200 mm deep, supports an eccentric prestressing force of 100 kN, the line of action of which coincides with the bottom kern of the section. The depth of the anchor plate is 50 mm.

$$P = 100 \text{ kN}$$

$$2y_{po} = 50 \text{ mm}$$

$$\text{Depth of the symmetric prism } 2y_o = 133 \text{ mm}$$

$$\therefore \text{ Distribution ratio } y_{po}/y_o = (50/133) = 0.375$$

$$\text{Position of zero stress} = (0.175) (2y_o) = 23.5 \text{ mm}$$

$$\text{Position of maximum stress} = (0.382) (2y_o) = 51 \text{ mm}$$

$$\text{Maximum tensile stress} = (0.285P/A) = 0.285 \times \left(\frac{100 \times 10^3}{100 \times 133} \right) = 2.13 \text{ N/mm}^2$$

∴ Total splitting tension

$$\begin{aligned} F_{bst} &= 0.3 P [1 - (y_{pv}/y_o)^{0.58}] \\ &= 0.3 \times 100 \times 10^3 [1 - (0.375)^{0.58}] = 13015 \text{ N} \end{aligned}$$

However, using the approximate formula for the area of stress diagram, Bursting tension = $(213 \times 110 \times 2.13) 100 = 15600 \text{ N}$

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EXAMPLE 10.6 Estimate the position and magnitude of the maximum transverse tensile stress and bursting tension for the end block with a concentric anchor force of 100 kN, as detailed in Example 10.1, using Rowe's method.

$$P_k = 100 \text{ kN}$$

$$2y_{po} = 50 \text{ mm}$$

$$2y_o = 100 \text{ mm}$$

$$\therefore \frac{y_{po}}{y_o} = 0.5$$

$$f_c = \left(\frac{100 \times 10^3}{100 \times 100} \right) = 10 \text{ N/mm}^2$$

$$\begin{aligned} f_{v(\max)} &= f_c [0.98 - 0.825 (y_{po}/y_o)] \\ &= 10[0.98 - 0.825 (0.5)] = 5.68 \text{ N/mm}^2 \end{aligned}$$

acting at a distance equal to $(0.5 \times 50) = 25 \text{ mm}$ from the end face. Bursting tension is given by,

$$F_{bst} = (100 \times 10^3)[(0.48 - 0.4 \times 0.5)] = 28000 \text{ N}$$

If the permissible tensile stress in concrete is assumed as 2 N/mm^2 , the corrected value of the bursting tension is,

$$F_{bst(\text{corrected})} = 28000 \left[1 - \left(\frac{2}{5.68} \right)^2 \right] = 24500 \text{ N}$$

EXAMPLE 10.7 The end block of a prestressed beam, 200 mm wide and 300 mm deep, has two Freyssinet anchorages (100 mm diameter) with their centres at 75 mm from the top and bottom of the beam. The force transmitted by each anchorage being 200 kN, estimate the maximum tensile stress and the bursting tension developed.

$$\text{Anchorage diameter} = 100 \text{ mm}$$

$$\text{Equivalent side of square } 2y_{po} = \sqrt{\frac{\pi}{4} \times 100^2} = 89 \text{ mm}$$

$$\text{Side of the surrounding prism } 2y_o = 150 \text{ mm}$$

$$\therefore \frac{y_{po}}{y_o} = 0.593$$

$$\text{Average compressive stress } f_c = \left(\frac{200 \times 10^3}{150 \times 150} \right) = 8.9 \text{ N/mm}^2$$

$$\text{Tensile stress } f_{v(\max)} = 8.9 [0.98 - 0.825(0.593)] = 4.45 \text{ N/mm}^2$$

$$\text{Transverse tension } F_{bst} = 200 \times 10^3 [0.48 - 0.4(0.593)] = 50000 \text{ N} = 50 \text{ kN}$$

Design of anchorage zone reinforcement

EXAMPLE 10.8 The end block of a post-tensioned prestressed concrete beam, 300 mm wide and 300 mm deep, is subjected to a concentric anchorage force of 832800 N by a Freyssinet anchorage of area 11720 mm². Design and detail the anchorage reinforcement for the end block.

Prestressing force, $P = 832800$ N

Average compressive stress,

$$f_c = \left(\frac{832800}{300 \times 300} \right) = 9.3 \text{ N/mm}^2$$

Anchorage diameter,

$$2y_{po} = \sqrt{\frac{11720 \times 4}{\pi}} = 123 \text{ mm}$$

Ratio,

$$\frac{y_{po}}{y_o} = \left(\frac{123}{300} \right) = 0.41$$

Tensile stress,

$$f_{v(\max)} = 9.3[0.98 - 0.825(0.41)] = 6 \text{ N/mm}^2$$

Bursting tension,
$$F_{bst} = 832800[0.48 - 0.4(0.41)] = 264000 \text{ N}$$

$$= 264 \text{ kN}$$

Using 10 mm diameter mild steel links with yield stress of 260 N/mm^2 ,

Number of bars required =
$$\frac{264000}{(0.87 \times 260 \times 79)} = 15$$

The reinforcement is to be arranged in the zone between $0.2y_o(0.2 \times 150) = 30 \text{ mm}$ and $y_o = 150 \text{ mm}$. The arrangement of reinforcement in the two perpendicular directions is shown in Fig. 10.19.

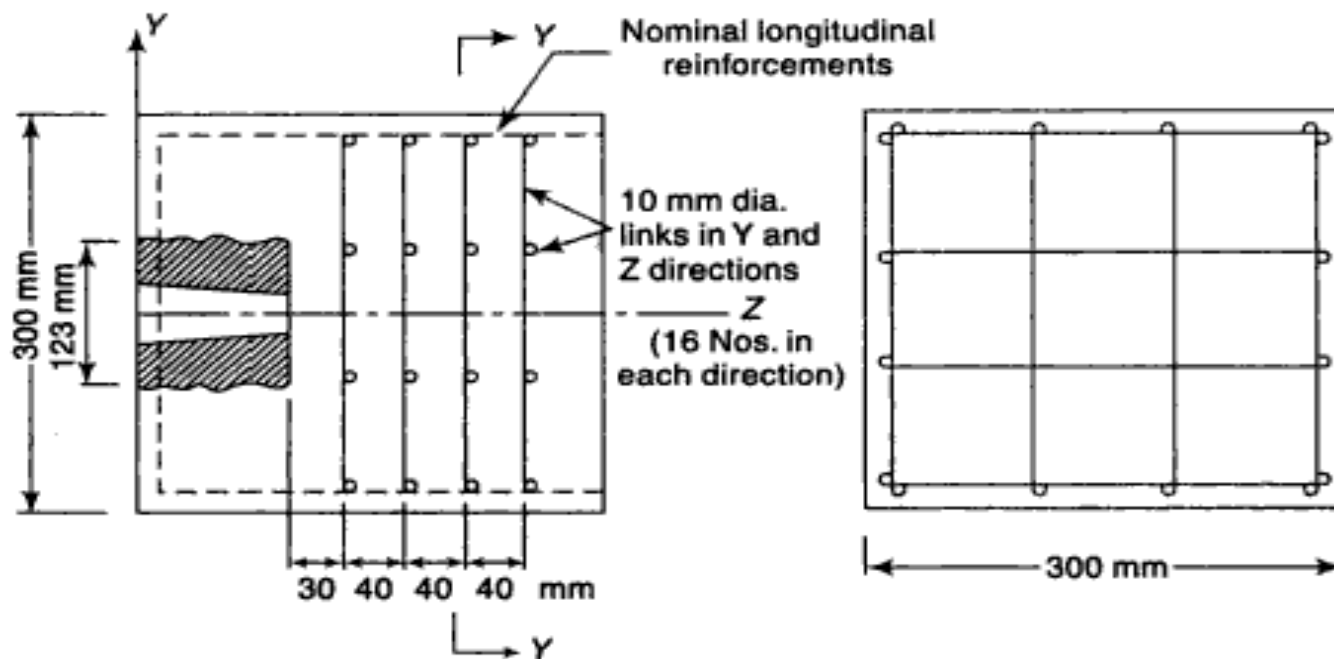


Fig. 10.19 Arrangement of Anchorage Zone Reinforcement

EXAMPLE 10.9 The end block of a post-tensioned prestressed member is 550 mm wide and 550 mm deep. Four cables, each made up of 7 wires of 12 mm diameter strands and carrying a force of 1000 kN, are anchored by plate anchorages, 150 mm by 150 mm, located with their centres at 125 mm from the edges of the end block. The cable duct is of 50 mm diameter. The 28-day cube strength of concrete f_{cu} is 45 N/mm^2 . The cube strength of concrete at transfer f_{ci} , is 25 N/mm^2 . Permissible bearing stresses behind anchorages should conform with IS: 1343. The characteristic yield stress in mild steel anchorage reinforcement is 260 N/mm^2 . Design suitable anchorages for the end block.

$$P_k = 1000 \text{ kN}$$

$$2y_{po} = 150 \text{ mm}$$

$$2y_o = 250 \text{ mm}$$

$$y_{po}/y_o = 0.6$$

$$\text{Area of the cable duct} = \left(\frac{\pi \times 50^2}{4} \right) = 2000 \text{ mm}^2$$

$$\text{Net area of the surrounding prism} = [(250)^2 - (2000)] = 60500 \text{ mm}^2$$

$$\text{Average compressive stress} \quad f_c = \left(\frac{1000 \times 10^3}{60500} \right) = 16.5 \text{ N/mm}^2$$

According to IS: 1343, the bearing stress shall not exceed $0.48 f_{ci} \sqrt{\frac{A_{br}}{A_{pun}}}$ or

$0.8 f_{ci}$, whichever is smaller,
where

A_{br} = bearing area

A_{pun} = punching area

$A_{br} = 60500 \text{ mm}^2$

$$A_{pun} = 22500 \text{ mm}^2$$

$$\therefore \frac{A_{br}}{A_{pun}} = 2.70$$

$$\begin{aligned} \text{Bearing stress limited to} &= 0.48 \times 25 \times \sqrt{2.70} = 19.7 \text{ N/mm}^2 \text{ or } (0.8 \times 25) \\ &= 20 \text{ N/mm}^2 \end{aligned}$$

$$\text{Actual bearing stress} = 16.5 \text{ N/mm}^2$$

Using Rowe–Zielinski method,

$$F_{v(\max)} = 16.5 [0.98 - 0.825 (0.6)] = 8 \text{ N/mm}^2$$

$$\text{Bursting tension } F_{bst} = 1000 [0.48 - 0.4(0.6)] = 240 \text{ kN}$$

Area of steel without considering the tensile strength of concrete,

$$A_s = \frac{(240 \times 10^3)}{(260 \times 0.87)} = 1060 \text{ mm}^2$$

Provide 10 bars of 12 mm diameter.

Limit state design criteria

Some of the important criteria concerning prestressed concrete for the ultimate limit state are given below.

1. Failure of one or more critical sections in flexure, shear, torsion, or due to their combinations.
2. bursting of prestressed concrete end blocks,
3. bearing failure at supports, anchorages or under-concentrated imposed loads,
4. bond and anchorage failure of reinforcement,
5. failure of connections between precast and cast *in situ* elements, and
6. failure due to elastic instability of members.